Way to Success

9

MATHS

Term- I

Prepared by

Mr. K. Dinesh M.Sc., M.Phil., P.G.D.C.A.,(Ph.D.,)

------For subject related clarifications------
Mail us: dinesh.skksv93@gmail.com & way2s100@gmail.com **Call us :** 7418865975 **Visit us:** www.waytosuccess.org
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CONTENT

SYMBOLS

$=$	equal to	$\neq$	not equal to
$<$	less than	$\leq$	less than or equal to
$>$	greater than	$\geq$	greater than or equal to
$\approx$	equivalent	$\cup$	union
$\cap$	intersection	$\in$	belongs to
$\notin$	does not belong to	$\subset$	proper subset of
$\subseteq$	subset of or is contained in	$\not\subset$	not a proper subset of
$\nsubseteq$	not a subset of or is not contained in	$A^{'}(or)A^{c}$	complement of A
$\emptyset\ (or)\{\ \}$	empty set or null set or void set	$P(A)$	power set of A
$n(A)$	number of elements in the set A	|||ly	similarly
$P(A)$	probability of the event A	Δ	symmetric difference
$\mathbb{N}$	natural numbers	$\mathbb{R}$	real numbers
$\mathbb{Z}$	integers	Δ	triangle
$\angle$	angle	$\perp$	perpendicular
$\parallel$	parallel to	$\Rightarrow$	implies
$\therefore$	therefore	$\lvert\ \rvert$	absolute value
$\cong$	congruent	$\equiv$	identically equal to
π	pi	$\pm$	plus or minus
$\blacksquare$	end of the proof	| (or) :	such that

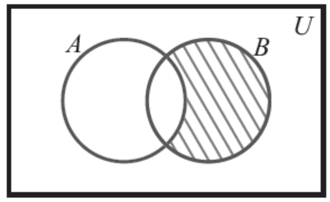
1. Theory of Sets

ONE MARK :

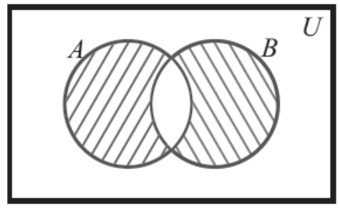
Choose the correct answer

1. If $A = \{5, \{5,6\}, 7\}$, which of the following is correct ?
   A) $\{\, \mathbf{5}, \mathbf{6} \,\} \in \boldsymbol{A}$ B) $\{\, 5 \,\} \in A$ C) $\{\, 7 \,\} \in A$ D) $\{\, 6 \,\} \in A$
2. If $X = \{a, \{b, c\}, d\}$, which of the following is a subset of X ?
   A) $\{\, a, b \,\}$ B) $\{\, b, c \,\}$ C) $\{\, c, d \,\}$ **D)** $\{\boldsymbol{a}, \boldsymbol{d}\,\}$
3. Which of the following statements are true?
   (i) For any set A, A is a proper subset of A
   (ii) For any set A, $\emptyset$ is a subset of A
   (iii) For any set A , A is a subset of A
   A) (i) and (ii) **B) (ii) and (iii)** C) (i) and (iii) D) (i), (ii) and (iii)
4. If a finite set A has m elements, then the number of non-empty proper subsets of A is
   A) 2^m B) $2^m - 1$ C) 2^{m-1} **D)** $\mathbf{2}(\, \mathbf{2}^{\boldsymbol{m}-\mathbf{1}} - \mathbf{1})$
5. The number of subsets of the set { 10, 11, 12 } is
   A) 3 **B) 8** C) 6 D) 7
6. Which one of the following is correct?
   A) $\{\, \boldsymbol{x}: \boldsymbol{x}^{\mathbf{2}} = -\mathbf{1}, \boldsymbol{x} \in \boldsymbol{Z}\} = \emptyset$ B) $\emptyset = 0$ C) $\emptyset = \{0\}$ D) $\emptyset = \{\emptyset\}$
7. Which one of the following is incorrect?
   A) Every subset of a finite set is finite
   B) $P = \{x: x - 8 = -8\}$ is a singleton set
   C) Every set has a proper subset
   D) Every non-empty set has at least two subsets, $\emptyset$ and the set itself
8. Which of the following is a correct statement?
   A) $\emptyset \subseteq \{\boldsymbol{a}, \boldsymbol{b}\}$ B) $\emptyset \in \{a, b\}$ C) $\{a\} \in \{a, b\}$ D) $a \subseteq \{a, b\}$
9. Which one of the following is a finite set?
   A) $\{x: x \in Z, x < 5\}$ B) $\{x: x \in W, x \geq 5\}$
   C) $\{x: x \in N, x > 10\}$ **D)** $\{\boldsymbol{x}: \boldsymbol{x}$ **is an even prime number**$\}$
10. Given $A = \{5, 6, 7, 8\}$. Which one of the following is incorrect?
   A) $\emptyset \subseteq A$ B) $A \subseteq A$ **C)** $\{\mathbf{7}, \mathbf{8}, \mathbf{9}\} \subseteq \boldsymbol{A}$ D) $\{\, 5 \,\} \subseteq A$
11. If $A = \{3,4,5,6\}$ and $B = \{1,2,5,6\}$ then $A \cup B =$
   A) { 1, 2 ,3, 4, 5, 6} B) { 1,2,3,4,6} C) {1, 2, 5, 6} D) {3,4,5,6}
12. The number of elements of the set $\{x: x \in Z, x^2 = 1\}$ is
   A)3 **B) 2** C) 1 D) 0
13. If $n(X) = m, n(Y) = n$ and $n(X \cap Y) = p$ then $n(X \cup Y) =$
   A) $m + n + p$ **B)** $\boldsymbol{m} + \boldsymbol{n} - \boldsymbol{p}$ C) $m - p$ D) $m - n + p$
14. If $U = \{1 ,2 ,3,4,5,6,7,8,9,10\}$ and $A = \{2,5,6,9,10\}$ then A' is
   A) { 2, 5,6,9,10} B) $\emptyset$ C) {1,3,5,10} **D)** $\{\mathbf{1}, \mathbf{3}, \mathbf{4}, \mathbf{7}, \mathbf{8}\}$

15. If $A \subseteq B$, then $A - B$ is
    A) B B) A **C) $\emptyset$** D) $B - A$
16. If A is a proper subset of B, then $A \cap B$ is
    A) $\boldsymbol{A}$ B) B
    C) $\emptyset$ D) $A \cup B$
17. If A is a proper subset of B , $A \cup B$
    A) A B) $\emptyset$ **C) $\boldsymbol{B}$** D) $A \cap B$
18. The shaded region in the adjoint diagram represents
    A) $A - B$ B) A'
    C) B' **D) $\boldsymbol{B - A}$**



19. If $A = \{a, b, c\}, B = \{e, f, g\}$, then $A \cap B =$
    A) $\emptyset$ B) A C) B D) $A \cup B$
20. The shaded region in the adjoining diagram represents
    A) $A - B$ B) $B - A$
    C) $\boldsymbol{A \triangle B}$ D) A'



Exercise 1.1

1. Which of the following are sets? Justify your answer

(i) The collection of good books -
Answer : Not a set (because the word good is not defined)

(ii) The collection of prime numbers less than 30
Answer : Set

(iii) The collection of ten most talented mathematics teachers
Answer : – Not a set (because the word most is not defined)

(iv) The collection of all students in your school
Answer : Set

(v) The collection of all even numbers
Answer : set

2. Let $\boldsymbol{A = \{0, 1, 2, 3, 4, 5\}}$. Inset the appropriate symbol $\in$ or $\notin$ in the blank spaces

(i) 0___A **Answer** : $0 \in A$
(ii) 6___A **Answer** : $6 \notin A$
(iii) 3___A **Answer** : $3 \in A$
(iv) 4 ___A **Answer** : $4 \in A$
(v) 7 ___A **Answer** : $7 \notin A$

3. Write the following sets in Set – Builder from

(i) The set of all positive even numbers
Answer : $\{x : x$ is a positive even number $\}$

(ii) The set of all whole numbers less than 20

Answer : $\{x : x \text{ is a whole number and } x < 20\}$

(iii) The set of all positive integers which are multiples of 3

Answer : $\{x : x \text{ is a positive integer and multiple of } 3\}$

(iv) The set of all odd natural numbers less than 15

Answer : $\{x : x \text{ is an odd natural number and } x < 15\}$

(v) The set of all leters in the word 'computer'

Answer : $\{x : x$ is a letter in the word 'computer' $\}$

4. Write the following sets in Roster form

(i) $A = \{x : x \in N, 2 < x \leq 10\}$ **Answer** : $A = \{3,4,5,6,7,8,9,10\}$

(ii) $B = \{x : x \in Z, -\frac{1}{2} < x < \frac{11}{2}\}$ **Answer** : $B = \{0, 1, 2, 3, 4, 5\}$

(iii) $C = \{x : x \text{ is a prime number and a divisor of } 6\}$ **Answer** : $C = \{2, 3\}$

(iv) $X = \{x : x = 2^n, n \in N \text{ and } n \leq 5\}$ **Answer** : $X = \{2, 4, 8, 16, 32\}$

(v) $M = \{x : x = 2y - 1, y \leq 5, y \in W\}$ **Answer** : $M = \{-1, 1, 3, 5, 7, 9\}$

(vi) $P = \{x : x \text{ is an integer}, x^2 \leq 16\}$ **Answer** : $P = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

5. Write the following sets in Descriptive form

(i) $A = \{a, e, i, o, u\}$

Answer : A is the set of all vowels in the English alphabet

(ii) $B = \{1,3,5,7,9,11\}$

Answer : B is the set of all odd natural numbers less than or equal to 11

(iii) $C = \{1, 4, 9, 16, 25\}$

Answer : C is the set of all square numbers less than 26

(iv) $P = \{x : x$ is a letter in the word 'set theory' $\}$

Answer : P is the set of all letters in the word 'set theory'

(v) $Q = \{x : x \text{ is a prime number between } 10 \text{ and } 20\}$

Answer : Q is the set of all prime numbers between 10 and 20

6. Find the cardinal number of the following sets

(i) $A = \{x : x = 5^n, n \in N \text{ and } n < 5\}$

Answer : $A = \{5^1, 5^2, 5^3, 5^4\}$

$n(A) = 4$

(ii) $B = \{x : x \text{ is a consonant in English Alphabet}\}$

Answer : Consonant in English Alphabet = { b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z}

$n(B) = 21$

(iii) $X = \{x : x \text{ is an even prime number}\}$

Answer : Even prime number = 2

$X = \{2\}$

$n(X) = 1$

(iv) $P = \{x : x < 0, x \in W\}$

Answer : $P = \{\emptyset\}$

$n(P) = 0$

(v) $Q = \{x: -3 \leq x \leq 5, x \in Z\}$

Answer : $Q = \{-3, -2, -1, 0, 1, 2, 3, 4,5\}$

$n(Q) = 9$

7. Identify the following sets as finite or infinite.

(i) $A = \{4, 5, 6, \dots\}$

Answer : Infinite set

(ii) $B = \{0, 1, 2, 3,4, \dots ,75\}$

Answer : Finite set

(iii) $X = \{x : x$ is an even natural number$\}$

Answer : $X = \{2, 4, 6, \dots \dots\}$

Infinite set

(iv) $Y = \{x : x$ is a multiple of 6 and $x > 0\}$

Answer : $Y = \{6 ,12, 18, \dots\}$

Infinite set

(v) $P =$ The set of letters in the word 'freedom'

Answer : $n(P) = 7$, Finite set

8. Which of the following sets are equivalent ?

(i) $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 3, 5, 7, 9\}$

Answer : $n(A) = 5, n(B) = 5$

A and B are equivalent

(ii) $X = \{x : x \in N, 1 < x < 6\}, Y\{x : x$ is a vowel in the English Alphabet $\}$

Answer : $X = \{2, 3, 4,5\}$, $Y = \{a, e, i, o, u\}$

$n(X) = 4, n(Y) = 5$

X and Y are not equivalent

(iii) $P = \{x : x$ is a prime number and $5 < x < 23\}$, $Q = \{x: x \in W, 0 \leq x < 5\}$

Answer : $P = \{7, 11, 13, 17, 19\}$, $Q = \{0, 1, 2, 3, 4\}$

$n(P) = 5, n(Q) = 5$

P and Q are equivalent

9. Which of the following sets are equal?

(i) $A = \{1, 2,3,4\}, B = \{4,3,2,1\}$

Answer : A and B have exactly the same elements. A and B are Equal

(ii) $A = \{4, 8, 12, 16\}, B = \{8, 4, 16, 18\}$

Answer : A and B are not equal

(iii) $X = \{2,4,6,8\}$

$Y = \{x: x$ is a positive even integer $0 < x < 10\}$

Answer :

$X = \{2,4,6,8\}$

$Y = \{2,4,6,8\}$

X and Y are Equal

(iv) $P = \{\ x : x\ is\ a\ multiple\ of\ 10, x \in N\}$

$Q = \{10,15,20,25,30, \dots\}$

Answer : $P = \{ 10, 20, 30, 40, \ldots \}$

$Q = \{10,15,20,25,30, \ldots \}$

P and Q are Not equal

10. From the sets given below, select equal sets.

$\boldsymbol{A = \{12, 14, 18, 22\}, B = \{11, 12, 13, 14\}, C = \{14, 18, 22, 24\},}$

$\boldsymbol{D = \{13, 11, 12, 14\}, E = \{-11, 11\}, F = \{10, 19\}, \ G = \{11, -11\}, H = \{10, 11\}}$

Answer : $B = \{11, 12, 13, 14\} = \{13, 11,12, 14\} = D$

$B = D$

$E = \{-11,11\} = \{11, -11\} = G$

$E = G$

11. Is $\emptyset = \{\emptyset\}$? Why?

Answer : No, $\emptyset$ contains no element but $\{\emptyset\}$ contains one element

12. Which of the sets are equal sets ? State the reason.

$\emptyset, \{0\}, \{\emptyset\}$

Answer : Each one is different from others

$\emptyset$ contains no element

$\{0\}$ contains one element

$\{\emptyset\}$ contains one element, i.e., the null set

13. Fill in the blank with $\subseteq$ or $\nsubseteq$ to make each statement true.

(i) {3} _____ {0, 2, 4, 6} **Answer** : $\{3\} \nsubseteq \{0, 2, 4, 6\}$

(ii) {a}____ {a, b, c} **Answer** : $\{a\} \subseteq \{a, b, c\}$

(iii) {8, 18} ___ {18, 8} **Answer** : $\{8, 18\} \subseteq \{18, 8\}$

(iv) {d}__ {a,b,c} **Answer** : $\{d\} \nsubseteq \{a,b,c\}$

14. Let $X = \{-3, -2, -1,0,1,2\}$ and $Y = \{x: x$ is an integer and $-3 \leq x < 2\}$

(i) Is X a subset of Y ? (ii) Is Y a subset of X?

Answer :

$X = \{-3, -2, -1,0,1,2\}$, $Y = \{-3, -2, -1,0,1\}$

i) X is not a subset of Y (ii) Y is a subset of X

15. Examine whether $\boldsymbol{A = \{x: x}$ is a positive integer divisible by 3 } is a subset of $\boldsymbol{B = \{x: x}$ is a multiple of 5, $\boldsymbol{x \in N\}}$

$A = \{3, 6,9,12,15,18, \ldots . \}$

$B = \{5, 10, 15,20,25, \ldots \}$

A is not a subset of B

16. Write down the power sets of the following sets.

(i) $A = \{x, y\}$

Answer :

$P(A) = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}$

(ii) $X = \{a, b, c\}$

Answer :

$P(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$

(iii) $A = \{5, 6, 7, 8\}$

Answer :

$P(A)$

$= \{\emptyset, \{5\}, \{6\}, \{7\}, \{8\}, \{5,6\}, \{5,7\}, \{5,8\}, \{6,7\}, \{6,8\}, \{7,8\}, \{5,6,7\}, \{5,6,8\}, \{5,7,8\}, \{6,7,8\}, \{5,6,7,8\}\}$

(iv) $A = \emptyset$

Answer :

$P(A) = \emptyset$

17. Find the number of subsets and the number of proper subsets of the following sets.

(i) $A = \{13, 14, 15, 16, 17, 18\}$

Answer :

$n(A) = 6$

The number of subsets $= n[P(A)] = 2^6 = 64$

The number of proper subsets $= 2^6 - 1 = 64 - 1 = 63$

(ii) $B = \{a, b, c, d, e, f, g\}$

Answer :

The number of subsets $= n[P(A)] = 2^7 = 128$

The number of proper subsets $= 2^7 - 1 = 128 - 1 = 127$

(iii) $X = \{x: x \in W, x \notin N\}$

Answer :

$X = \{0\}$

The number of subsets $= n[P(A)] = 2^1 = 2$

The number of proper subsets $= 2^1 - 1 = 2 - 1 = 1$

18. (i) If $A = \emptyset$, find $n[P(A)]$

Answer :

A contains no element

The number of subsets $= n[P(A)] = 2^0 = 1$

(ii) If $n(A) = 3$, find $n[P(A)]$

Answer :

The number of subsets $= n[P(A)] = 2^3 = 8$

(iii) If $n[P(A)] = 512$ find $n(A)$

Answer :

$n[P(A)] = 512 = 2^9$

$n(A) = 9$

(iv) If $n[P(A)] = 1024$ find $n(A)$

Answer :

$n[P(A)] = 1024 = 2^{10}$

$n(A) = 10$

19. If $n[P(A)] = 1$, What can you say about the set A ?

Answer :

$n[P(A)] = 1 = 2^0$

A is the empty set

20. **Let $A = \{x : x \text{ is a natural number} < 11\}$**

$B = \{x : x \text{ is an even number and } 1 < x < 21\}$

$C = \{x : x \textit{ is an integer and } 15 \le x \le 25\}$

(i) List the elements of A, B, C

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$B = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$C = \{15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25\}$

(ii) Find $n(A), n(B), n(C)$

$$n(A) = 10,$$
$$n(B) = 10,$$
$$n(C) = 11$$

(iii) State whether the following are True or False

(a) $7 \in B$ - False

(b) $16 \notin A$ - True

(c) $\{15, 20, 25\} \subset C$ - True

(d) $\{10, 12\} \subset B$ - True

Exercise 1 . 2

1. **Find $A \cup B$ and $A \cap B$ for the following sets.**

(i) $A = \{0, 1, 2, 4, 6\}$ and $B = \{-3, -1, 0, 2, 4, 5\}$

(ii) $A = \{2, 4, 6, 8\}$ and $B = \emptyset$

(iii) $A = \{x : x \in N, x \le 5\}$ and $B = \{x : x \text{ is a prime number less than } 11\}$

(iv) $A = \{x : x \in N, 2 < x \le 7\}$ and $B = \{x : x \in W, 0 \le x \le 6\}$

Answer :

(i) $A = \{0, 1, 2, 4, 6\}$ and $B = \{-3, -1, 0, 2, 4, 5\}$

$A \cup B = \{0, 1, 2, 4, 6\} \cup \{-3, -1, 0, 2, 4, 5\} = \{-3, -1\ 0, 1, 2, 4, 5, 6\}$

$A \cap B = \{0, 1, 2, 4, 6\} \cap \{-3, -1, 0, 2, 4, 5\} = \{0, 2, 4\}$

(ii) $A = \{2, 4, 6, 8\}$ and $B = \emptyset$

$$A \cup B = \{2, 4, 6, 8\} \cup \emptyset = \{2, 4, 6, 8\}$$
$$A \cap B = \{2, 4, 6, 8\} \cap \emptyset = \emptyset$$

(iii) $A = \{x : x \in N, x \le 5\}$ and $B = \{x : x \text{ is a prime number less than } 11\}$

$$A = \{1, 2, 3, 4, 5\}, B = \{2, 3, 5, 7\}$$
$$A \cup B = \{1, 2, 3, 4, 5\} \cup \{2, 3, 5, 7\} = \{1, 2, 3, 4, 5, 7\}$$
$$A \cap B = \{1, 2, 3, 4, 5\} \cap \{2, 3, 5, 7\} = \{2, 3, 5\}$$

(iv) $A = \{x : x \in N, 2 < x \le 7\}$ and $B = \{x : x \in W, 0 \le x \le 6\}$

$$A = \{3, 4, 5, 6, 7\}, B = \{0, 1, 2, 3, 4, 5, 6\}$$
$$A \cup B = \{3, 4, 5, 6, 7\} \cup \{0, 1, 2, 3, 4, 5, 6\} = \{0, 1, 2, 3, 4, 5, 6, 7\}$$
$$A \cap B = \{3, 4, 5, 6, 7\} \cap \{0, 1, 2, 3, 4, 5, 6\} = \{3, 4, 5, 6\}$$

2. **If $A = \{x : x \text{ is a multiple of } 5, x \le 30 \text{ and } x \in N\}$, $B = \{1, 3, 7, 10, 12, 15, 18, 25\}$**

Find (i) $A \cup B$ (ii) $A \cap B$

Answer :

$A = \{5, 10, 15, 20, 25, 30\}, B = \{1, 3, 7, 10, 12, 15, 18, 25\}$

(i) $A \cup B = \{5, 10, 15, 20, 25, 30\} \cup \{1, 3,7,10,12,15,18,25\} = \{1, 3, 5, 7,10, 12,15, 18,20, 25, 30\}$

(ii) $A \cap B = \{5, 10, 15, 20, 25, 30\} \cap \{1, 3,7,10,12,15,18,25\} = \{10, 15, 25\}$

3. **If $X = \{x: x = 2n, x \le 20 \text{ and } n \in N\}$ and $Y = \{x: x = 4n, x \le 20 \text{ and } n \in W\}$**
Find (i) $X \cup Y$ (ii) $X \cap Y$

Answer :

$X = \{2, 4, 6,8, 10, 12, 14,16,18,20\}, Y = \{0, 4, 8, 12, 16, 20\}$

(i) $X \cup Y = \{2, 4, 6,8, 10, 12, 14,16,18,20\} \cup \{0, 4, 8, 12, 16, 20\}$
$= \{0, 2, 4, 6, 8,10,12,14, 16, 18,20\}$

(ii) $X \cap Y = \{2, 4, 6,8, 10, 12, 14,16,18,20\} \cup \{0, 4, 8, 12, 16, 20\}$
$= \{4, 8, 12, 16, 20\}$

4. $\boldsymbol{U = \{1, 2, 3, 6, 7, 12, 17, 21, 35, 52, 56\}}$
$\boldsymbol{P = \{\text{numbers divisible by } 7\}, Q = \{\text{Prime numbers}\}}$
List the elements of the set $\{x: x \in P \cap Q\}$

Answer :

$U = \{1, 2,3 ,6,7,12,17,21,35,52,56\}$
$P = \{7, 21, 35, 56\}$
$Q = \{2, 3, 7, 17\}$
$P \cap Q = \{7, 21, 35, 56\} \cap \{2, 3, 7, 17\} = \{7\}$

5. **State which of the following sets are disjoint**
(i) $A = \{2, 4, 6, 8\}; B = \{x: x \text{ is an even number} < 10, x \in N\}$
(ii) $X = \{1, 3, 5, 7, 9\}, Y = \{0, 2, 4, 6, 8, 10\}$
(iii) $P = \{x: x \text{ is a prime} < 15\}$
$Q = \{x: x \text{ is a multiple of 2 and } x < 16\}$
(iv) $R = \{a, b, c, d, e\}, S = \{d, e, a, b, c\}$

Answer :

(i) $A = \{2,4,6,8\}; B = \{x: x \text{ is an even number} < 10, x \in N\}$
$A = \{2,4,6,8\}; B = \{2, 4,6, 8\}$
$A \cap B = \{2, 4,6, 8\} \ne \emptyset$
A and B are said to be overlapping sets

(ii) $X = \{1,3,5,7,9\}, Y = \{0,2,4,6,8,10\}$
$X \cap Y = \{1,3,5,7,9\} \cap \{0,2,4,6,8,10\} = \emptyset$
X and Y are disjoint sets.

(iii) $P = \{x: x \text{ is a prime} < 15\}$
$Q = \{x: x \text{ is a multiple of 2 and } x < 16\}$
$P = \{2, 3, 5, 7, 11,13\}$
$Q = \{ 2, 4, 6, 8,10, 12,14\}$
$P \cap Q = \{2, 3, 5, 7, 11,13\} \cap \{ 2, 4, 6, 8,10, 12,14\} = \{2 \} \ne \emptyset$
P and Q are said to be overlapping sets

(iv) $R = \{a, b, c, d, e\}, S = \{d, e, a, b, c\}$
$R \cap S = \{a, b, c, d, e\} \cap \{d, e, a, b, c\} = \{a, b, c, d, e\} \ne \emptyset$
R and S are said to be overlapping sets

6. **(i) If $\boldsymbol{U = \{x: 0 \le x \le 10, x \in W\}}$ and $\boldsymbol{A = \{x: x \text{ is a multiple of } 3\}}$, find $\boldsymbol{A'}$**

(ii) If $\boldsymbol{U}$ is the set of natural numbers and $\boldsymbol{A'}$ is the set of all composite numbers , then what is $\boldsymbol{A}$?

Answer :

(i) $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{3, 6, 9\}$

$A' = U - A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{3, 6, 9\} = \{ 0,1,2,4,5,7,8,10\}$

(ii) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \dots \}$

$A' = \{4, 6, 8,9, \dots \}$

$A = U - A' = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \dots \} - \{4, 6, 8,9, \dots \} = \{1, 2, 3, 5,7, \dots \}$

A is the set of all prime numbers and 1

7. **If $\boldsymbol{U = \{a, b, c, d, e, f, g, h\}}$, $\boldsymbol{A = \{a, b, c, d\}}$ and $\boldsymbol{B = \{b, d, f, g\}}$**

Find (i) $\boldsymbol{A \cup B}$ (ii) $\boldsymbol{(A \cup B)'}$ (iii) $\boldsymbol{A \cap B}$ (iv) $\boldsymbol{(A \cap B)'}$

Answer :

$U = \{a, b, c, d, e, f, g, h\}, A = \{a, b, c, d\} , B = \{b, d, f, g\}$

(i) $A \cup B = \{a, b, c, d\} \cup \{b, d, f, g\} = \{a, b, c, d, f, g\}$

(ii) $(A \cup B)' = U - A = \{a, b, c, d, e, f, g, h\} - \{a, b, c, d, f, g\} = \{e, h\}$

(iii) $A \cap B = \{a, b, c, d\} \cap \{b, d, f, g\} = \{b, d\}$

(iv) $(A \cap B)' = U - (A \cap B) = \{a, b, c, d, e, f, g, h\} - \{b, d\} = \{a, c, e, f, g, h \}$

8. **If $\boldsymbol{U = \{x: 1 \le x \le 10, x \in N\}}$, $\boldsymbol{A = \{1, 3, 5, 7, 9\}}$ and $\boldsymbol{B = \{2, 3, 5, 9, 10\}}$**

Find (i) $\boldsymbol{A'}$ (ii) $\boldsymbol{B'}$ (iii) $\boldsymbol{A' \cup B'}$ (iv) $\boldsymbol{A' \cap B'}$

Answer :

$U = \{1, 2, 3, 4, 5,6, 7, 8, 9, 10\}, A = \{1, 3, 5, 7, 9\} , B = \{2,3,5,9,10\}$

(i) $A' = U - A = \{1, 2, 3, 4, 5,6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\} = \{ 2, 4, 6, 8,10\}$

(ii) $B' = U - B = \{1, 2, 3, 4, 5,6, 7, 8, 9, 10\} - \{2,3,5,9,10\} = \{1, 4, 6, 7, 8\}$

(iii) $A' \cup B' = \{ 2, 4, 6, 8\} \cup \{1, 4, 6, 7, 8\} = \{1, 2, 4, 6, 7, 8,10\}$

(iv) $A' \cap B' = \{ 2, 4, 6, 8\} \cap \{1, 4, 6, 7, 8\} = \{4, 6, 8\}$

9. **Given that $\boldsymbol{U = \{3, 7, 9, 11, 15, 17, 18\}}$, $\boldsymbol{M = \{3, 7, 9, 11\}}$ and $\boldsymbol{N = \{7, 11, 15, 17\}}$ find**

(i) $\boldsymbol{M - N}$ (ii) $\boldsymbol{N - M}$ (iii) $\boldsymbol{N' - M}$ (iv) $\boldsymbol{M' - N}$

(v) $\boldsymbol{M \cap (M - N)}$ (vi) $\boldsymbol{N \cup (N - M)}$ (vii) $\boldsymbol{n(M - N)}$

Answer :

$U = \{3,7,9,11,15,17,18\}, M = \{3,7,9,11\} , N = \{7,11,15,17\}$

(i) $M - N = \{3,7,9,11\} - \{7,11,15,17\} = \{3, 9\}$

(ii) $N - M = \{7,11,15,17\} - \{3,7,9,11\} = \{15, 17 \}$

(iii) $N' - M$

$N' = U - N = \{3,7,9,11,15,17,18\} - \{7,11,15,17\} = \{3, 9, 18\}$

$N' - M = \{3, 9, 18\} - \{3,7,9,11\} = \{18\}$

(iv) $M' - N$

$M' = U - M = \{3,7,9,11,15,17,18\} - \{3,7,9,11\} = \{15, 17, 18\}$

$M' - N = \{15, 17, 18\} - \{7,11,15,17\} = \{18\}$

(v) $M \cap (M - N) = \{3,7,9,11\} \cap \{3, 9\} = \{3, 9\}$

(vi) $N \cup (N - M) = \{7,11,15,17\} \cup \{15, 17 \} = \{7, 11, 15, 17\}$

(vii) $n(M - N) = 2$

10. **If $A = \{3, 6, 9, 12, 15, 18\}, B = \{4, 8, 12, 16, 20\}, C = \{2, 4, 6, 8, 10, 12\}$ and $D = \{5, 10, 15, 20, 25\}$ find (i) $A - B$ (ii) $B - C$ (iii) $C - D$ (iv) $D - A$ (v) $n(A - C)$**

Answer :

(i) $A - B = \{3, 6,9,12,15,18\} - \{4,8,12,16,20\} = \{3,6,9,15,18\}$

(ii) $B - C = \{4,8,12,16,20\} - \{2,4,6,8,10,12\} = \{16, 20\}$

(iii) $C - D = \{2,4,6,8,10,12\} - \{5, 10,15,20,25\} = \{2,4,6,8,12\}$

(iv) $D - A = \{5, 10,15,20,25\} - \{3, 6,9,12,15,18\} = \{5, 10, 20, 25\}$

(v) $n(A - C)$

$A - C = \{3, 6,9,12,15,18\} - \{2,4,6,8,10,12\} = \{3,9,15,18\}$

$n(A - C) = 4$

11. **Let $U = \{x: x$ is a positive integer less than 50 $\}$, $A = \{x: x$ divisible by 4$\}$ and $B = \{x: x$ leaves a remainder 2 when divided by 14$\}$**

(i) List the elements of U, A and B

(ii) Find $A \cup B, A \cap B, n(A \cup B), n(A \cap B)$

Answer :

(i) $U = \{1, 2, 3, \ldots 49\}$

$A = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48\}$

$B = \{16, 30, 44\}$

(ii) $A \cup B = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48\} \cup \{16, 30, 44\}$

$= \{4, 8, 12, 16, 20, 24, 28, 30,32, 36, 40, 44, 48\}$

$A \cap B = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48\} \cap \{16, 30, 44\} = \{16, 44\}$

$n(A \cup B) = 13$

$n(A \cap B) = 2$

12. **Find the symmetric difference between the following sets.**

(i) $X = \{a, d, f, g, h\}, Y = \{b, e, g, h, k\}$

(ii) $P = \{x: 3 < x < 9, x \in N\}, Q = \{x: x < 5, x \in W\}$

(iii) $A = \{-3, -2, 0, 2, 3, 5\}, B = \{-4, -3, -1, 0, 2, 3\}$

Answer :

(i) $X = \{a, d, f, g, h\}, Y = \{b, e, g, h, k\}$

$X - Y = \{a, d, f, g, h\} - \{b, e, g, h, k\} = \{a, d, f\}$

$Y - X = \{b, e, g, h, k\} - \{a, d, f, g, h\} = \{b, e, k\}$

$X \Delta Y = (X - Y) \cup (Y - X) = \{a, b, d, e, f, k\}$

(ii) $P = \{x: 3 < x < 9, x \in N\}, Q = \{x: x < 5, x \in W\}$

$P = \{4, 5, 6, 7, 8\}, \; Q = \{0, 1, 2, 3, 4\}$

$P - Q = \{4, 5, 6, 7, 8\} - \{0, 1, 2, 3, 4\} = \{5, 6, 7, 8\}$

$Q - P = \{0, 1, 2, 3, 4\} - \{4, 5, 6, 7, 8\} = \{0, 1, 2, 3\}$

$P \Delta Q = (P - Q) \cup (Q - P) = \{5, 6, 7, 8\} \cup \{0, 1, 2, 3\} = \{0, 1, 2, 3, 5, 6, 7, 8\}$

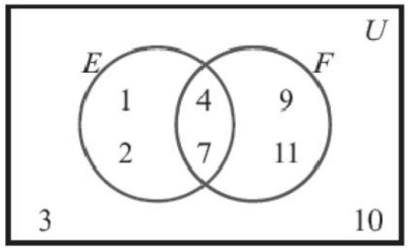
(iii) $A = \{-3, -2, 0, 2, 3, 5\}, B = \{-4, -3, -1, 0, 2, 3\}$

$A - B = \{-3, -2, 0, 2, 3, 5\} - \{-4, -3, -1, 0, 2, 3\} = \{-2, 5\}$

$B - A = \{-4, -3, -1, 0, 2, 3\} - \{-3, -2, 0, 2, 3, 5\} = \{-4, -1\}$

$A \Delta B = (A - B) \cup (B - A) = \{-2, 5\} \cup \{-4, -1\} = \{-4, -2, -1, 5\}$

13. Use the Venn diagram to answer the following questions



(i) List the elements of $U, E, F, E \cup F$ and $E \cap F$

(ii) Find $n(U), n(E \cup F)$ and $n(E \cap F)$

Answer :

(i) $U = \{1, 2, 3, 4, 7, 9, 10, 11\}, E = \{1, 2, 4, 7\}, F = \{4, 7, 9, 11\}$

$E \cup F = \{1, 2, 4, 7\} \cup \{4, 7, 9, 11\} = \{1, 2, 4, 7, 9, 11\}$

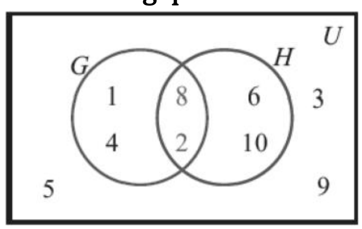
$E \cap F = \{1, 2, 4, 7\} \cap \{4, 7, 9, 11\} = \{4, 7\}$

(ii) $n(U) = 8$

$n(E \cup F) = 6$

$n(E \cap F) = 2$

14. Use the Venn diagram to answer the following questions



(i) List U, G and H

(ii) Find $G', H', G' \cap H', n(G \cup H)'$ and $n(G \cap H)'$

Answer :

(i) $U = \{1, 2, 3, 4, 5, 6, 8, 9, 10\}, G = \{1, 2, 4, 8\}, H = \{2, 6, 8, 10\}$

(ii) $G' = U - G = \{1, 2, 4, 5, 6, 8, 9, 10\} - \{1, 2, 4, 8\} = \{3, 5, 6, 9, 10\}$

$H' = \{1, 2, 3, 4, 5, 6, 8, 9, 10\} - \{2, 6, 8, 10\} = \{1, 3, 4, 5, 9\}$

$G' \cap H' = \{3, 5, 6, 9, 10\} \cap \{1, 3, 4, 5, 9\} = \{3, 5, 9\}$

$G \cup H = \{1, 2, 4, 8\} \cup \{2, 6, 8, 10\} = \{1, 2, 4, 6, 8, 10\}$

$(G \cup H)' = U - (G \cup H) = \{1, 2, 3, 4, 5, 6, 8, 9, 10\} - \{1, 2, 4, 6, 8, 10\}$

$= \{3, 5, 9\}$

$n(G \cup H)' = 3$

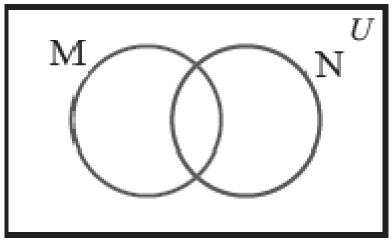
$G \cap H = \{1, 2, 4, 8\} \cap \{2, 6, 8, 10\} = \{2, 8\}$

$(G \cap H)' = U - (G \cap H) = \{1, 2, 3, 4, 5, 6, 8, 9, 10\} - \{2, 8\} = \{1, 3, 4, 5, 6, 9, 10\}$

$n(G \cap H)' = 7$

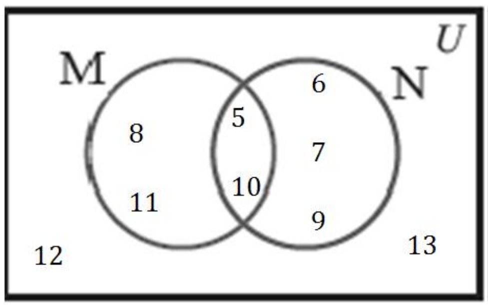
Exercise 1. 3

1. **Place the elements of the following sets in the proper location on the given Venn diagram**



$U = \{5, 6, 7, 8, 9, 10, 11, 12, 13\}, M = \{5\ , 8, 10, 11\}, N = \{5, 6, 7\}$

Answer :



2. **If A and B are two sets such that A has 50 elements, B has 65 elements and $A \cup B$ has 100 elements, how many elements does $A \cap B$ have?**

Answer :

$n(A) = 50, n(B) = 65, n(A \cup B) = 100,\ \ n(A \cap B) = ?$

By using the formula

$n(A \cap B) = n(A) + n(B) - n(A \cup B) = 50 + 65 - 100 = 115 - 100 = 15$

3. **If A and B are two sets containing 13 and 16 elements respectively, then find the minimum and maximum number of elements in $A \cup B$ have?**

Answer :

$n(A) = 13, n(B) = 16$

Maximum number of element

$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 13 + 16 - 0 = 29$ If $A \cap B = \emptyset$

Minimum number of element

$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 13 + 16 - 13 = 16$, If $n(A \cap B) = 13$

4. **If $\boldsymbol{n(A \cap B) = 5, n(A \cup B) = 35, n(A) = 13}$, find $\boldsymbol{n(B)}$**

Answer :

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$35 = 13 + n(B) - 5$$
$$n(B) = 35 - 13 + 5 = 27$$

5. **If $n(A) = 26, n(B) = 10, n(A \cup B) = 30, n(A') = 17$, find $n(A \cap B)$ and $n(U)$**

Answer :

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$30 = 26 + 10 - n(A \cap B)$

$n(A \cap B) = 36 - 30 = 6$

$n(U) = n(A) + n(A') = 26 + 17 = 43$

6. **If $n(U) = 38, n(A) = 16, n(A \cap B) = 12, n(B') = 20$, find $n(A \cup B)$**

Answer :

$n(U) = n(B) + n(B')$

$n(B) = n(U) - n(B') = 38 - 20 = 18$

$n(A \cup B) = 16 + 18 - 12 = 34 - 12 = 22$

7. **Let A and B be two finite sets such that $n(A - B) = 30, n(A \cup B) = 180$. Find $n(B)$**

Answer :

$$n(B) = n(A \cup B) - n(A - B) = 180 - 30 = 150$$

8. **The population of a town is 10000. Out of these 5400 persons read newspaper A and 4700 read newspaper B. 1500 persons read both the newspapers. Find the number of persons who do not read either of the two papers .**

Answer :

The population of a town is 10000 , $n(U) = 10000$

5400 persons read newspaper A, $n(A) = 5400$

4700 read newspaper B, $n(B) = 4700$

1500 persons read both the newspapers $n(A \cap B) = 1500$

$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 5400 + 4700 - 1500 = 8600$

The number of persons who do not read either of the two papers

$n(A \cup B)' = 10000 - 8600 = 1400$

9. **In a school, all the students play either Foot ball or Volley ball or both. 300 students play Foot ball, 270 students play Volley ball and 120 students play both games. Find**

(i) the number of students who play Foot ball only

(ii) the number of students who play Volley ball only

(iii) the total number of students in the school

Answer :

300 students play Foot ball,

270 students play Volley ball

120 students play both games.

(i) The number of students who play Foot ball only $= 300 - 120 = 180$

(ii) The number of students who play Volley ball only $= 270 - 120 = 150$

(iii) The total number of students in the school $= 300 + 270 - 120 = 450$

10. **In an examination 150 students secured first class in English or Mathematics. 115 students secured first class in Mathematics. How many students secured first class in English only?**

Answer :

$n(M) + n(E) = 150$

$n(M) = 115$

$n(M) + n(E) = 150$

$n(E) = 150 - n(M) = 150 - 115 = 35$

11. In a group of 30 persons, 18 take tea. Find how many take coffee but not tea, if each persons takes at least one of the drinks.

Answer :

Total number of persons $n(U) = 30$

18 persons take tea. $n(T) = 18$

Persons take coffee $n(C) = n(U) - n(T) = 30 - 18 = 12$

12. In a village there are 60 families. Out of these 28 families speak only Tamil and 20 families speak only Urudu. How many families speak both Tamil and Urudu.

Answer :

In a village there are 60 families.

28 families speak only Tamil, $n(T) = 28$

20 families speak only Urudu, $n(U) = 20$

$n(T \cup U) = n(T) + n(U) - n(T \cap U)$

$60 = 28 + 20 - n(T \cap U)$

$n(T \cap U) = 60 - 48 = 12$

13. In a school 150 students passed X standard examination. 95 students applied for Group I and 82 students applied for Group II in the Higher secondary course. If 20 students applied neither of the two, how many students applied for both groups ?

Answer :

Total number of students 150

95 students applied for Group I

82 students applied for Group II

20 students applied neither of Group I and Group II

Applied for both groups $= x$

Group I alone $= 95 - x$

Group II alone $= 82 - x$

$95 - x + x + 82 - x = 150 - 20$

$95 + 82 - x = 130$

$177 - x = 130$

$177 - 130 = x$

$x = 47$

$\therefore$ The students applied for both the groups =47

14. Pradeep is a section Chief for an electric utility company. The employees in his section cut down tall trees or climb poles. Pradeep recently reported the following information to the management of the utility. Out of 100 employees in my section, 55 can cut tall trees, 50 can climb poles, 11 can do both, 6 can't do any of the two. Is this information correct ?

Answer :

$T =$ set of employees who cut all the trees

$P =$ Set of employees who climb the poles

$n(T) = 55, n(P) = 50, n(T \cap P) = 11$

$n(T \cup P) = n(T) + n(P) - n(T \cap P)$

$= 55 + 50 - 11$

$= 105 - 11$

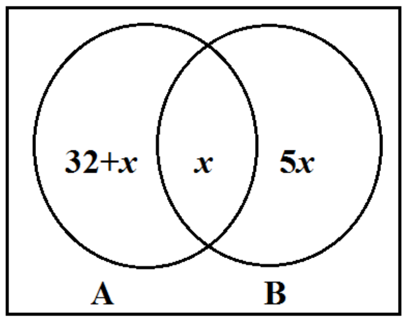
$= 94$

The number of employees who cannot do anything $= 100 - 94 = 6$

Yes, the given information is correct

15. **A and B are two sets such that $n(A - B) = 32 + x, n(B - A) = 5x$ and $n(A \cap B) = x$ Illustrate the information by means of a Venn diagram. Given that $n(A) = n(B)$. Calculate (i) the value of x (ii) $n(A \cup B)$**

Answer :



(i) Given that

$n(A) = n(B)$

$32 + x + x = 5x + x$

$32 + 2x = 6x$

$32 = 6x - 2x$

$32 = 4x$

$x = 8$

(ii) $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

$= 32 + x + x + 5x$

$= 32 + 7x$

$= 32 + 7(8)$

$= 32 + 56$

$= 88$

16. **The following table shows the percentage of the students of a school who participated in Elocution and Drawing competitions.**

Competition	Elocution	Drawing	Both
Percentage of Students	55	45	20

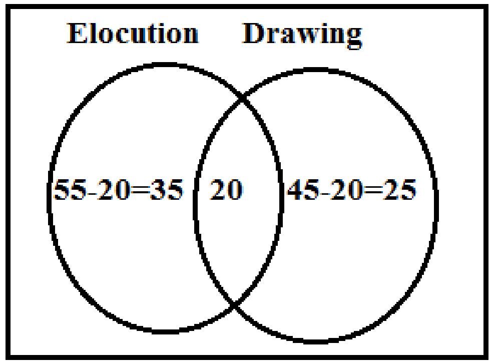
Draw a Venn diagram to represent this information and use it to find the percentage of the students who

(i) participated in Elocution only

(ii) participated in Drawing only

(iii) did not participate in any one of the competitions.

Answer :



(i) Participated in Elocution only $= 55 - 20 = 35\%$
(ii) Participated in Drawing only $= 45 - 20 = 25\%$
(iii) Did not participate in any one of the competitions. $= 100 - 35 - 20 - 25$
$= 100 - 80$
$= 20\%$

17. A village has total population of 2500 people. Out of which 1300 people use brand. A soap and 1050 people use brand B soap and 250 people use both brands. Find the percentage of population who use neither or these soaps.

Answer :

Total population of village = 2500
Number of peoples using brand A, $n(A) = 1300$
Number of peoples using brand B , $n(B)= 1050$
Number of peoples using both, $n(A \cap B) = 250$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 1300 + 1050 - 250$$
$$= 2350 - 250$$
$$= 2100$$

population who use neither or these soaps $= 2500 - 2100 = 400$

The percentage of population who use neither or these soaps $= \frac{400}{2500} \times 100 = 16\%$

2. Real number system

ONE MARK :

Choose the correct answer

1. A number having non-terminating and recurring decimal expansion is

   A) an integer **B) a rational number**

   C) an irrational number D) a whole number

2. If a number has a non-terminating and non-recurring decimal expansion, then it is

   A) a rational number B) a natural number **C) an irrational number**

   D) an integer

3. Decimal form of $-\frac{3}{4}$ is

   A) $-\mathbf{0.75}$ B) -0.50 C) -0.25 D) -0.125

4. The $\frac{p}{q}$ form of $0.\overline{3}$ is

   A) $\frac{1}{7}$ B) $\frac{2}{7}$ **C) $\frac{1}{3}$** D) $\frac{2}{3}$

5. Which one of the following is not true?

   A) Every natural number is a rational number

   B) Every real number is a rational number

   C) Every whole number is a rational number

   D) Every integer is a rational number

6. Which one of the following has a terminating decimal expansion?

   A) $\frac{5}{52}$ B) $\frac{7}{9}$ C) $\frac{8}{15}$ D) $\frac{1}{12}$

7. Which one of the following is an irrational number?

   A) π B) $\sqrt{9}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$

8. Which of the following are irrational numbers?

   (i) $\sqrt{2+\sqrt{3}}$ (ii) $\sqrt{4+\sqrt{25}}$ (iii) $\sqrt[3]{5+\sqrt{7}}$ (iv) $\sqrt{8-\sqrt[3]{8}}$

   A) (ii), (iii) and (iv) B) (i),(ii) and (iv)

   C) (i), (ii) and (iii) **D) (i), (iii) and (iv)**

Exercise 2.1

1. State whether the following statements are true or false.

(i)	Every natural number is a whole number	**Answer** : True
(ii)	Every whole number is a natural number	**Answer** : False
(iii)	Every integer is a rational number	**Answer** : True
(iv)	Every rational number is a whole number	**Answer** : False
(v)	Every rational number is an integer	**Answer** : False
(vi)	Every integer is a whole number	**Answer** : False

2. Is zero a rational number? Give reasons for your answer.

Yes, for $0 = \frac{0}{1} = \frac{0}{2} = \frac{0}{3} = \frac{0}{-1} = \cdots$

3. Find any two rational numbers between $-\frac{5}{7}$ and $-\frac{2}{7}$

A rational number between $-\frac{5}{7}$ and $-\frac{2}{7} = \frac{1}{2}\left(-\frac{5}{7}-\frac{2}{7}\right) = \frac{1}{2}\left(-\frac{7}{7}\right) = \frac{1}{2}(-1) = -\frac{1}{2}$

Another rational number between $-\frac{1}{2}$ and $-\frac{2}{7} = \frac{1}{2}\left(-\frac{1}{2}-\frac{2}{7}\right) = \frac{1}{2}\left(\frac{-7-4}{14}\right) = \frac{1}{2}\left(\frac{-11}{14}\right)$

Exercise 2.2

1. Convert the following rational numbers into decimals and state the kind of decimal expansion

(i) $\frac{42}{100}$ **(ii)** $8\frac{2}{7}$ **(iii)** $\frac{13}{55}$ **(iv)** $\frac{459}{500}$ **(v)** $\frac{1}{11}$ **(vi)** $-\frac{3}{13}$ **(vii)** $\frac{19}{3}$ **(viii)** $-\frac{7}{32}$

(i) $\frac{42}{100} = 0.42$, terminating

(ii) $8\frac{2}{7} = \frac{8(7)+2}{7}$

$= \frac{56+2}{7}$

$= \frac{58}{7}$

$= 8.\overline{285714}$,

nonterminating and recurring

$$
\begin{array}{r|l}
 & 8.285714.. \\
7 & 58 \\
 & \underline{56} \\
 & \quad 20 \\
 & \quad \underline{14} \\
 & \qquad 60 \\
 & \qquad \underline{56} \\
 & \qquad\quad 40 \\
 & \qquad\quad \underline{35} \\
 & \qquad\qquad 50 \\
 & \qquad\qquad \underline{49} \\
 & \qquad\qquad\quad 10 \\
 & \qquad\qquad\quad \underline{7} \\
 & \qquad\qquad\qquad 30 \\
 & \qquad\qquad\qquad \underline{28} \\
 & \qquad\qquad\qquad\quad 2 \\
 & \qquad\qquad\qquad\quad \ddots
\end{array}
$$

(iii) $\frac{13}{55} = 0.\overline{236}$

Non- terminating and recurring

$$
\begin{array}{r|l}
 & 0.236 \\
55 & 130 \\
 & \underline{110} \\
 & \quad 200 \\
 & \quad \underline{165} \\
 & \qquad 350 \\
 & \qquad \underline{330} \\
 & \qquad\quad 2 \\
 & \qquad\quad \ddots
\end{array}
$$

(iv) $\frac{459}{500} = 0.918$, terminating

$$\begin{array}{r|l} & 0.918 \\ \hline 500 & 4590 \\ & \underline{4500} \\ & 900 \\ & \underline{500} \\ & 4000 \\ & \underline{4000} \\ & 0 \end{array}$$

(v) $\frac{1}{11} = 0.\overline{09}$, non-terminating and recurring

$$\begin{array}{r|l} & 0.09 \\ \hline 11 & 100 \\ & \underline{99} \\ & 1 \\ & \ddots \end{array}$$

(vi) $-\frac{3}{13} = -0.\overline{230769}$, non- terminating and recurring

$$\begin{array}{r|l} & 0.230769 \\ \hline 13 & 30 \\ & \underline{26} \\ & 40 \\ & \underline{39} \\ & 100 \\ & \underline{91} \\ & 90 \\ & \underline{78} \\ & 120 \\ & \underline{117} \\ & 3 \end{array}$$

(vii) $\frac{19}{3} = 6.\overline{3}$, non-terminating and recurring

$$\begin{array}{r|l} & 6.3 \\ \hline 3 & 19 \\ & \underline{18} \\ & 10 \\ & \underline{9} \\ & 1 \end{array}$$

(viii) $-\frac{7}{32} = -0.21875$, terminaing

$$\begin{array}{r|l} & 0.21875 \\ \hline 32 & 70 \\ & \underline{64} \\ & 60 \\ & \underline{32} \\ & 280 \\ & \underline{256} \\ & 240 \\ & \underline{224} \\ & 160 \\ & \underline{160} \\ & 0 \end{array}$$

2. Without actual division, find which of the following rational numbers have terminating decimal expansion

(i) $\frac{5}{64}$ (ii) $\frac{11}{12}$ (iii) $\frac{27}{40}$ (iv) $\frac{8}{35}$

(i) $\frac{5}{64}$

$64 = 2^5$

$\frac{5}{64} = \frac{5}{2^5 \times 5^0}$, So, $\frac{5}{64}$ has terminating

(ii) $\frac{11}{12}$

$12 = 2^2 \times 3$

$\frac{11}{12} = \frac{11}{2^2 \times 3}$

Since it is not in the form $\frac{p}{2^m \times 5^n}$, $\frac{11}{12}$ has a non-terminating

(iii) $\frac{27}{40}$

$40 = 2^3 \times 5^1$

$\frac{27}{40} = \frac{27}{2^3 \times 5^1}$. So, $\frac{27}{40}$ has a terminating

(iv) $\frac{8}{35}$

$\frac{8}{35} = \frac{8}{5^1 \times 7}$

Since it is not in the form $\frac{p}{2^m \times 5^n}$, $\frac{8}{35}$ has a non-terminating

3. Express the following decimal expansions into rational numbers

(i) $0.\overline{18}$

Let $x = 0.\overline{18}$. Then $x = 0.18181818 \ldots$

Since two digits are repeating, Multiplying both sides by 100, we get

$100x = 18.181818 \ldots. = 18 + 0.181818 \ldots. = 18 + x$

$100x - x = 18$

$99x = 18$

$x = \frac{2}{11}$

(ii) $0.\overline{427}$

Let $x = 0.\overline{427}$. Then $x = 0.427427427 \ldots.$

Since three digits are repeating, multiplying both sides by 1000, we get

$1000x = 427.427427427 \ldots. = 427 + 0.427427 \ldots.. = 427 + x$

$1000x - x = 427$

$999x = 427$

$x = \frac{427}{999}$

(iii) $0.\overline{0001}$

Let $x = 0.\overline{0001}$. Then $x = 0.000100010001$ ....

Since four digits are repeating, multiplying both sides by 10000, we get

$10000x = 1.00010001 \ldots$

$= 1 + 0.00010001 \ldots$

$= 1 + x$

$10000x - x = 1$

$9999x = 1$

$x = \frac{1}{9999}$

(iv) $1.\overline{45}$

Let $x = 1.\overline{45}$. Then $x = 1.454545 \ldots$

Since two digits are repeating, multiplying both sides by 100, we get

$100x = 145.454545 + \cdots = 144 + 1.454545 \ldots = 144 + x$

$100x - x = 144$

$x = \frac{144}{99} = \frac{16}{11}$

(v) $7.\overline{3}$

Let $x = 7.\overline{3}$, then 7.3333

Multiplying both sides by 10, we get

$10x = 73.3333333333$

$= 66 + 7.3333$

$= 66 + x$

$9x = 66$

$x = \frac{66}{9} = \frac{22}{3}$

(vi) $0.4\overline{16}$

Let $x = 0.4\overline{16}$, Then $x = 0.41616161616$ ....

Since two digits are repeating, multiplying both sides by 1000, we get

$1000x = 416.161616$ ....

$= 412 + 4.16161616 \ldots$

$= 412 + 10(0.4161616 \ldots)$

$= 412 + 10x$

$1000x - 10x = 412$

$990x = 412$

$x = \frac{412}{990} = \frac{206}{495}$

4. Express $\frac{1}{13}$ in decimal form. Find the number of digits in the repeating block.

$\frac{1}{13} = 0.\overline{076923}$

The number of digits in the repeating block is 6

$$
\begin{array}{r|l}
 & 0.076923 \\
13 & 100 \\
 & \underline{91} \\
 & \quad 90 \\
 & \quad \underline{78} \\
 & \quad 120 \\
 & \quad \underline{117} \\
 & \qquad 30 \\
 & \qquad \underline{26} \\
 & \qquad 40 \\
 & \qquad \underline{39} \\
 & \qquad\quad 100 \\
 & \qquad\quad 91
\end{array}
$$

5. Find the decimal expansions of $\frac{1}{7}$ and $\frac{2}{7}$ by division method. Without using the long division method, deduce the decimal expression of $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ from the decimal expansion of $\frac{1}{7}$.

$\frac{1}{7} = 0.\overline{142857}$ $\frac{2}{7} = 0.\overline{285714}$

$$
\begin{array}{r|l}
 & 0.142857 \\
7 & 10 \\
 & \underline{7} \\
 & \ 30 \\
 & \ \underline{28} \\
 & \quad 20 \\
 & \quad \underline{14} \\
 & \qquad 60 \\
 & \qquad \underline{56} \\
 & \qquad\ 40 \\
 & \qquad\ \underline{35} \\
 & \qquad\quad 50 \\
 & \qquad\quad \underline{49} \\
 & \qquad\qquad 1
\end{array}
\qquad
\begin{array}{r|l}
 & 0.285714 \\
7 & 20 \\
 & \underline{14} \\
 & \ 60 \\
 & \ \underline{56} \\
 & \quad 40 \\
 & \quad \underline{35} \\
 & \qquad 50 \\
 & \qquad \underline{49} \\
 & \qquad\ 10 \\
 & \qquad\ \underline{7} \\
 & \qquad\quad 30 \\
 & \qquad\quad \underline{28} \\
 & \qquad\qquad 2
\end{array}
$$

$\frac{3}{7} = 0.\overline{142857} + 0.\overline{285714} = 0.\overline{428571}$

$\frac{4}{7} = 0.\overline{142857} + 0.\overline{428571} = 0.\overline{571428}$

$\frac{5}{7} = 0.\overline{142857} + 0.\overline{571428} = 0.\overline{714285}$

$\frac{6}{7} = 0.\overline{142857} + 0.\overline{714285} = 0.\overline{857142}$

Exercise 2.3

1. Locate $\sqrt{5}$ on the number line.

Draw a number line. Mark points O and E on the number line such that O represents the number zero and E represents the number $\sqrt{3}$

$\therefore OE = \sqrt{3}$ unit. Draw $EF \perp OE$ such that $EF = 1$ unit. Join OF

In right triangle OEF, by Pythagorean theorem

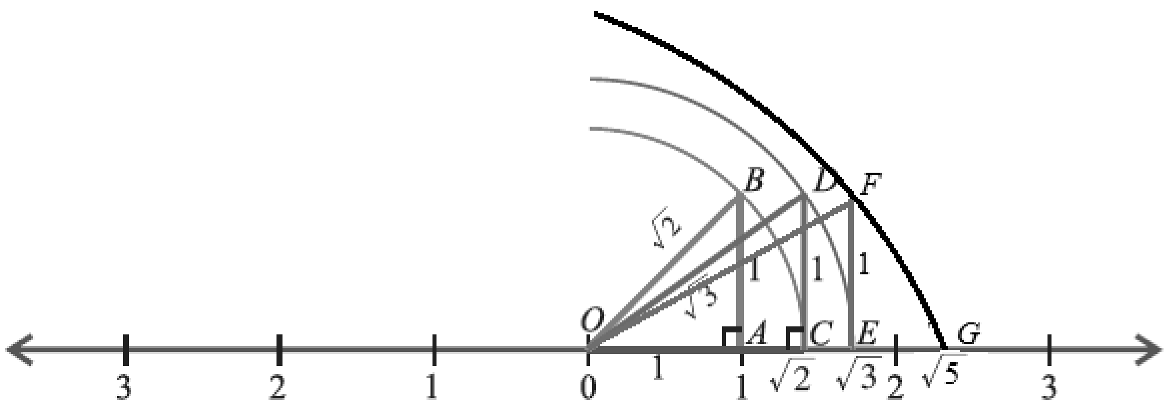
$OF^2 = OE^2 + EF^2$

$= \left(\sqrt{3}\right)^2 + 2 = 3 + 2 = 5$

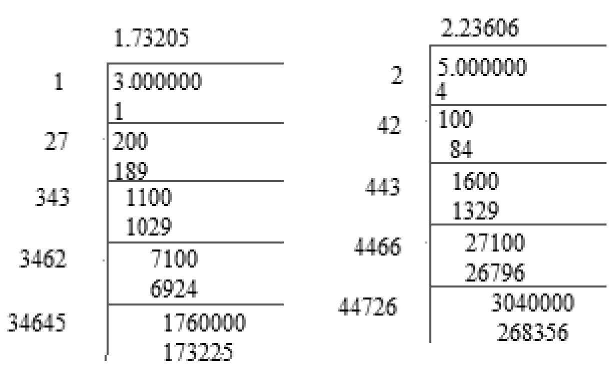
$OD = \sqrt{5}$

With O as centre and radius OF, draw an arc to intersect the number line at G on the right side of O. Clearly $OG = OF = \sqrt{5}$

Thus, E represents $\sqrt{5}$ on the number line.



2. Find any three irrational numbers between $\sqrt{3}$ and $\sqrt{5}$



We find out three irrational numbers between $\sqrt{3}$ and $\sqrt{5}$

The three numbers whose decimal expansions are non terminating and recurring

Infact the numbers are 1.83205…, 1.93205…,2.03205……

3. Find any two irrational numbers between 3 and 3.5

To find two irrational numbers between 3 and3.5

The two numbers whose decimal expansions are non terminating and recurring.

Infact there are infinitely many such numbers

The two numbers are 3.1011001110001111….

3.2022002220002222…

4. Find any two irrational numbers between 0.15 and 0.16

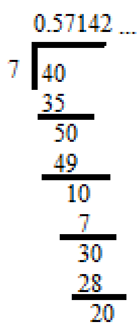
To find irrational numbers between 0.15 and 0.16

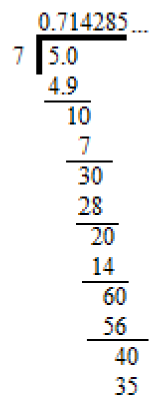
The two numbers whose decimal expansions are non terminating and recurring.

Infact the numbers are 0.1510100110001110…

0.153030033000333…

5. Insert any two irrational numbers between $\frac{4}{7}$ and $\frac{5}{7}$





To find two irrational numbers between $\frac{4}{7}$ and $\frac{5}{7}$. We find two numbers whose decimal expansions are non-terminating and non-recurring. Infact, there are infinitely many such numbers. Two such numbers are 0.58088008880…, 0.59099009990…

6. Find any two irrational numbers between $\sqrt{3}$ and 2.

To find two irrational numbers between $\sqrt{3}$ and 2. We find two numbers whose decimal expansions are non-terminating and recurring. Infact, there are infinitely many such numbers. Two such numbers are 1.83205…, 1.93205…

7. Find a rational number and also an irrational number between 1.1011001110001… and 2.1011001110001..

One rational number: 1.102,

An irrational number: 1.9199119991119…

8. Find any two rational numbers between 0.12122122212222… and 0.2122122212222…

Two rational numbers are 0.13, 0.20

Exercise 2.4

1. Using the process of successive magnification

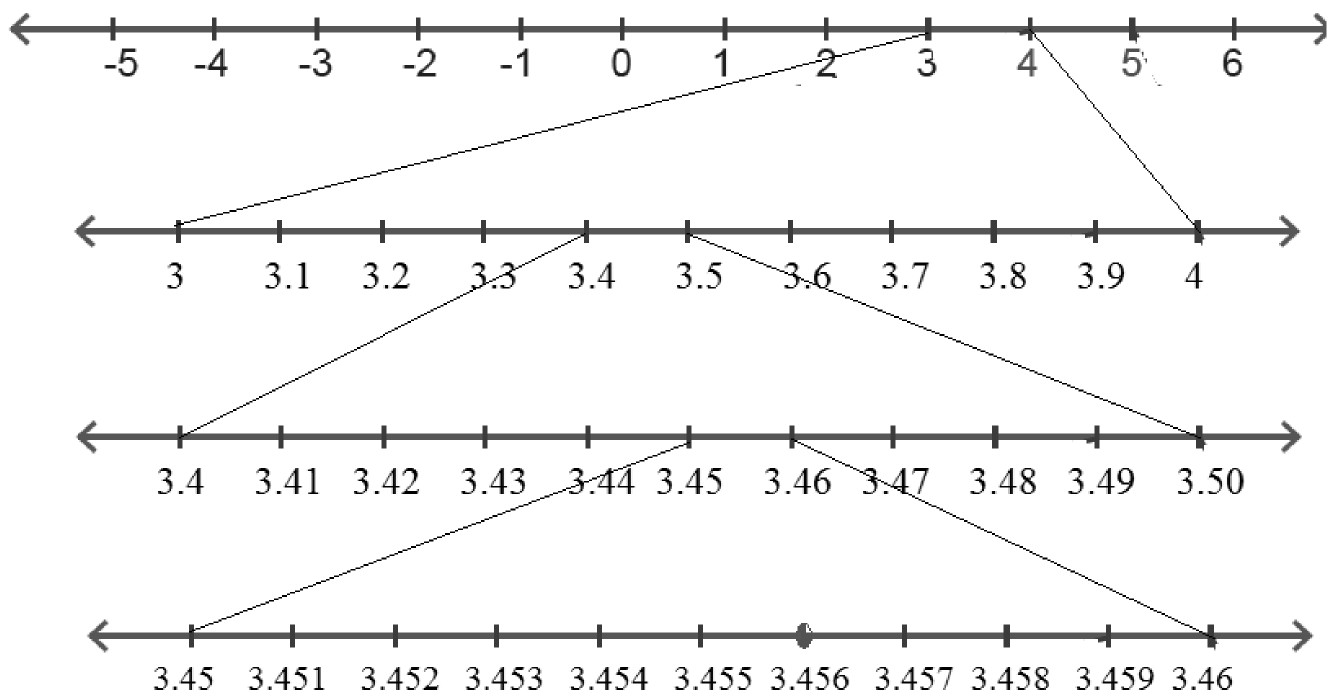
(i) Visualise 3.456 on the number line

Step I: First we note that 3.456 lies between 3 and 4

Step II: Divide the portion between 3 and 4 into 10 equal parts and use a magnifying glass to visualize that 3.456 lies between 3.4 and 3.5

Step III: Divide the portion between 3.4 and 3.5 into 10 equal parts and use a magnifying glass to visulaise that 3.456 lies between 3.45 and 3.46

Step IV: Divide the portion 3.45 and 3.46 into 10 equal parts and use magnifying glass to visualize that 3.456 lies between 3.455 and 3.457



(ii) Visualise $6.7\overline{3}$ on the number line, upto 4 decimal places

Visualise $6.7\overline{3}$ on the number line, upto 4 decimal places, that is 6.7333

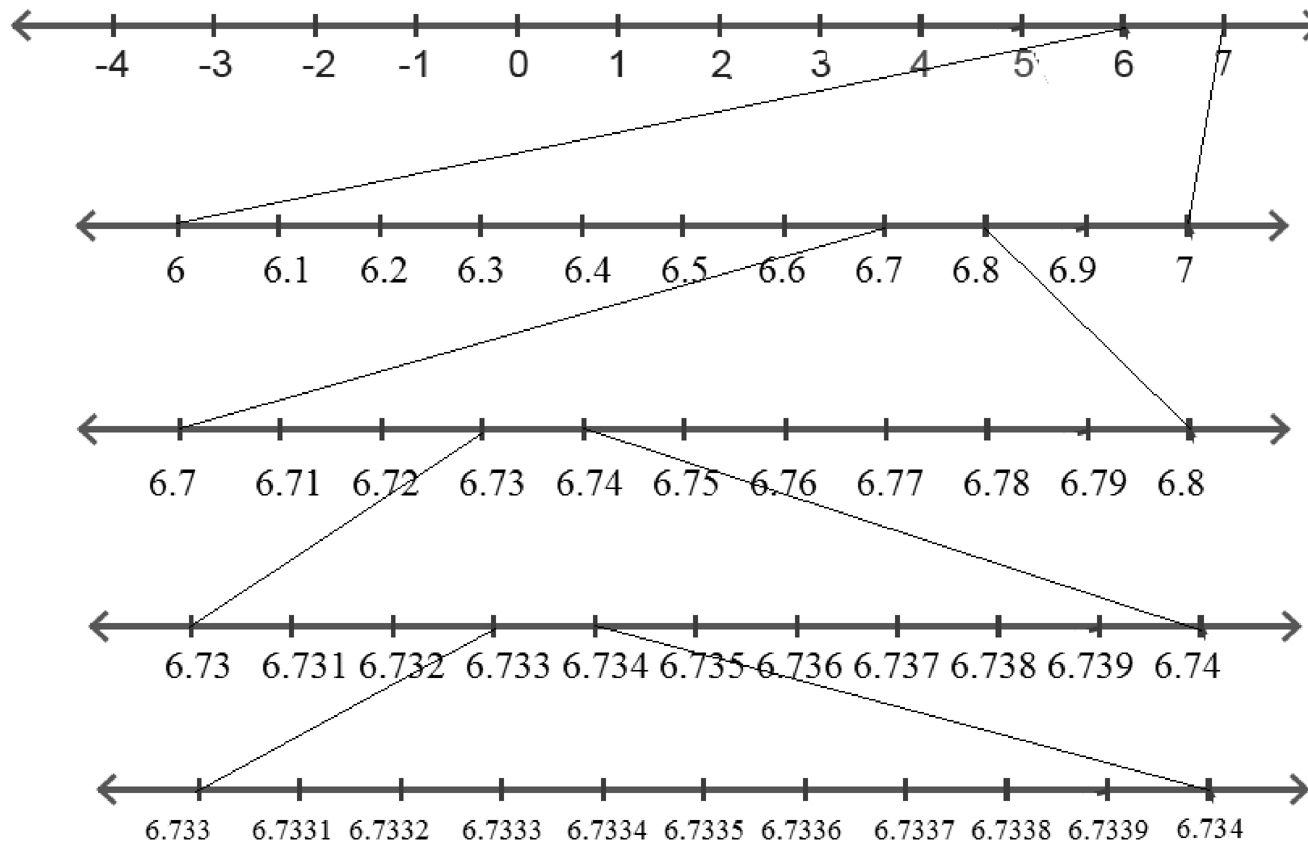
Step I: First we note that 6.7333 lies between 6 and 7

Step II: Divide the portion between 6 and 7 into 10 equal parts and use a magnifying glass to visualize that 6.7333 lies between 6.7 and 6.8

Step III: Divide the portion between 6.7 and 6.8 into 10 equal parts and use a magnifying glass to visulaise that 6.7333 lies between 6.73 and 6.74

Step IV: Divide the portion 6.73 and 6.74 into 10 equal parts and use magnifying glass to visualize that 6.7333 lies between 6.733 and 6.734

Step IV: Divide the portion 6.733 and 6.734 into 10 equal parts and use magnifying glass to visualize that 6.7333 lies between 6.7332 and 6.7334



3. Algebra

ONE MARK :

Choose the correct answer

1. The coefficients of x^2 and x in $2x^3 - 3x^2 - 2x + 3$ are respectively

   A) 2, 3 **B) $-3, -2$** C) $-2, -3$ D) $2, -3$

2. The degree of the polynomial $4x^2 - 7x^3 + 6x + 1$ is

   A) 2 B) 1 **C) 3** D) 0

3. The polynomial $3x - 2$ is a

   A) linear polynomial B) quadratic polynomial
   C) cubic polynomial D) constant polynamial

4. The polynomial $4x^2 + 2x - 2$ is a

   A) linear polynomial **B)quadratic polynomial**
   C) cubic polynomial D) constant polynomial

5. The zero of the polynomial $2x - 5$ is

   A) $\frac{5}{2}$ B) $-\frac{5}{2}$ C) $\frac{2}{5}$ D) $-\frac{2}{5}$

6. The root of the polynomial equation $3x - 1 = 0$ is

   A) $x = -\frac{1}{3}$ **B) $x = \frac{1}{3}$** C) $x = 1$ D) $x = 3$

7. The roots of the polynomial equation $x^2 + 2x = 0$ are

   A) $x = 0, 2$ B) $x = 1, 2$ C) $x = 1, -2$ **D) $x = 0, -2$**

8. If a polynomial $p(x)$ is divided by $(ax + b)$, then the remainder is

   A) $p\left(\frac{b}{a}\right)$ **B) $p\left(-\frac{b}{a}\right)$** C) $p\left(\frac{a}{b}\right)$ D) $p\left(-\frac{a}{b}\right)$

9. If the polynomial $x^3 - ax^2 + 2x - a$ is divided $(x - a)$, then remainder is

   A) a^3 B) a^2 **C) a** D) $-a$

10. If $(ax - b)$ is a factor of $p(x)$, then

    A) $p(b) = 0$ B) $p\left(-\frac{b}{a}\right) = 0$ C) $p(a) = 0$ **D) $p\left(\frac{b}{a}\right) = 0$**

11. One of the factors of $x^2 - 3x - 10$ is

    A) $x - 2$ B) $x + 5$ **C) $x - 5$** D) $x - 3$

12. One of the factors of $x^3 - 2x^2 + 2x + 1$ is

    A) $x - 1$ B) $x + 1$ C) $x - 2$ D) $x + 2$

Exercise 3.1

1. State whether the following expressions are polynomials in one variable or not. Give reasons for your answer.

(i) $2x^5 - x^3 + x - 6$ (ii) $3x^2 - 2x + 1$ (iii) $y^3 + 2\sqrt{3}$

(iv) $x - \frac{1}{x}$ (v) $\sqrt[3]{t} + 2t$ (vi) $x^3 + y^3 + z^6$

Answer :

(i) $2x^5 - x^3 + x - 6$

Polynomial in one variable

(ii) $3x^2 - 2x + 1$

Polynomial one variable

(iii) $y^3 + 2\sqrt{3}$

Polynomial in one variable

(iv) $x - \frac{1}{x}$

Since the exponent of x is not a whole number is not a polynomial

(v) $\sqrt[3]{t} + 2t$

Since the exponent of t is not a whole number is not a polynomial

(vi) $x^3 + y^3 + z^6$

Polynomial in three variables.

2. Write the coefficient of x^2 and x in each of the following

(i) $2 + 3x - 4x^2 + x^3$ (ii)$\sqrt{3}\,x + 1$ (iii) $x^3 + \sqrt{2}\,x^2 + 4x - 1$ (iv) $\frac{1}{3}\,x^2 + x + 6$

Answer :

(i) $2 + 3x - 4x^2 + x^3$

Coefficient of $x^2 = -4$

Coefficient of $x = 3$

(ii) $\sqrt{3}\,x + 1$

Coefficient of $x^2 = 0$

Coefficient of $x = \sqrt{3}$

(iii) $x^3 + \sqrt{2}\,x^2 + 4x - 1$

Coefficient of $x^2 = \sqrt{2}$

Coefficient of $x = 4$

(iv) $\frac{1}{3}\,x^2 + x + 6$

Coefficient of $x^2 = \frac{1}{3}$

Coefficient of $x = 1$

3. Write the degree of each of the following polynomials

(i) $4 - 3x^2$ (ii) $5y + \sqrt{2}$ (iii) $12 - x + 4x^3$ (iv) 5

Answer :

(i) $4 - 3x^2$, Degree 2

(ii) $5y + \sqrt{2}$, Degree 1

(iii) $12 - x + 4x^3$, Degree 3

(iv) 5 , Degree 0

4. Classify the following polynomials based on their degree.

(i) $3x^2 + 2x + 1$ (ii) $4x^3 - 1$ (iii) $y + 3$ (iv) $y^2 - 4$ (v) $4x^3$ (vi) $2x$

Answer :

(i) $3x^2 + 2x + 1$

Quadratic polynomial, since the highest degree of the variable is two.

(ii) $4x^3 - 1$

Cubic polynomial, since the highest degree of the variable is three.

(iii) $y + 3$

Linear polynomial, since the highest degree of the variable is one.

(iv) $y^2 - 4$

Quadratic polynomial, since the highest degree of the variable is two.

(v) $4x^3$

Cubic polynomial, since the highest degree of the variable is three.

(vi) $2x$

Linear polynomial, since the highest degree of the variable is one.

5. Give one example of a binomial of degree 27 and monomial of degree 49 and trinomial of degree 36.

Example of Binomial of degree 27 : $ax^{27} + b$

Example of Monomial of degree 49 : cx^{49}

Example of Trinomial of degree 36 : $lx^{36} + mx^{35} + nx^{2}$

Exercise 3.2

1. Find the zeros of the following polynomials.

(i) $p(x) = 4x - 1$ (ii) $p(x) = 3x + 5$ (iii) $p(x) = 2x$ (iv) $p(x) = x + 9$

Answer :

(i) $p(x) = 4x - 1$

Given that $p(x) = 4x - 1 = 4\left(x - \frac{1}{4}\right)$

$p\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) - 1 = 1 - 1 = 0$

Hence $\frac{1}{4}$ is the zero of $p(x)$

(ii) $p(x) = 3x + 5$

Given that $p(x) = 3x + 5 = 3\left(x + \frac{5}{3}\right)$

$p\left(-\frac{5}{3}\right) = 3\left(-\frac{5}{3}\right) + 5 = -5 + 5 = 0$

Hence $\frac{5}{3}$ is a zero of $p(x)$

(iii) $p(x) = 2x$

$p(x) = 2(0) = 0$

(iv) $p(x) = x + 9$

$p(-9) = -9 + 9 = 0$

2. Find the roots of the following polynomial equations.

(i) $x - 3 = 0$ (ii) $5x - 6 = 0$ (iii) $11x + 1 = 0$ (iv) $-9x = 0$

Answer :

(i) $x - 3 = 0$

Given that $x - 3 = 0 \Rightarrow x = 3$

$\therefore x = 3$ is a root of $x - 3 = 0$

(ii) $5x - 6 = 0$

Given that $5x - 6 = 0 \Rightarrow 5x = 6 \Rightarrow x = \frac{6}{5}$

$\therefore x = \frac{6}{5}$ is a root of $5x - 6 = 0$

(iii) $11x + 1 = 0$

Given that $11x + 1 = 0$

$11x = -1$

$x = -\frac{1}{11}$

$\therefore x = -\frac{1}{11}$ is a root of $11x + 1 = 0$

(iv) $-9x = 0$

Given that $-9x = 0 \Rightarrow x = 0$

$\therefore x = 0$ is a root of $-9x = 0$

3. Verify whether the following are roots of the polynomial equations indicated against them

(i) $x^2 - 5x + 6 = 0; x = 2, 3$ (ii) $x^2 + 4x + 3 = 0, x = -1, 2$

(iii) $x^3 - 2x^2 - 5x + 6 = 0, x = 1, -2, 3$

(iv) $x^3 - 2x^2 - x + 2 = 0$; $x = -1, 2, 3$

Answer :

(i) $x^2 - 5x + 6 = 0; x = 2, 3$

$p(x) = x^2 - 5x + 6$

$p(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = -6 + 6 = 0$

Hence $x = 2$ is a root of $x^2 - 5x + 6 = 0$

$p(3) = 3^2 - 5(3) + 6 = 9 - 15 + 6 = 15 - 15 = 0$

Hence $x = 3$ is a root of $x^2 - 5x + 6 = 0$

(ii) $x^2 + 4x + 3 = 0, x = -1, 2$

$p(x) = x^2 + 4x + 3$

$p(-1) = (-1)^2 + 4(-1) + 3 = 1 - 4 + 3 = 0$

Hence $x = -1$ is a root of $x^2 + 4x + 3 = 0$

$p(2) = 2^2 + 4(2) + 3 = 4 + 8 + 3 = 15 \neq 0$

Hence $x = 2$ is not a root of $x^2 + 4x + 3 = 0$

(iii) $x^3 - 2x^2 - 5x + 6 = 0, x = 1, -2, 3$

$p(x) = x^3 - 2x^2 - 5x + 6$

$p(1) = 1^3 - 2(1)^2 - 5(1) + 6 = 1 - 2 - 5 + 6 = 7 - 7 = 0$

Hence $x = 1$ is a root of $x^3 - 2x^2 - 5x + 6$

$p(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6 = -8 - 8 + 10 + 6 = -16 + 16 = 0$

Hence $x = -2$ is a root of $x^3 - 2x^2 - 5x + 6$

$p(3) = 3^3 - 2(3)^2 - 5(3) + 6 = 27 - 2(9) - 15 + 6 = 27 - 18 - 15 + 6 = 33 - 33 = 0$

Hence $x = 3$ is a root of $x^3 - 2x^2 - 5x + 6$

(iv) $x^3 - 2x^2 - x + 2 = 0$; $x = -1, 2, 3$

$p(x) = x^3 - 2x^2 - x + 2$

$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2 = -1 - 2 + 1 + 2 = 0$

Hence $x = -1$ is a root of $x^3 - 2x^2 - x + 2 = 0$

$p(2) = 2^3 - 2(2)^2 - 2 + 2 = 8 - 8 - 2 + 2 = 0$

Hence $x = 2$ is a root of $x^3 - 2x^2 - x + 2 = 0$

$p(3) = 3^3 - 2(3)^2 - 3 + 2 = 27 - 18 - 3 + 2 = 8 \neq 0$

Hence $x = 3$ is not a root of $x^3 - 2x^2 - x + 2 = 0$

Exercise 3.3

1. Find the quotient and the remainder of the following division

1. $\mathbf{(5x^3 - 8x^2 + 5x - 7) \div (x - 1)}$

$$
\begin{array}{r|l}
 & 5x^2 - 3x + 2 \\
\hline
x - 1 & 5x^3 - 8x^2 + 5x - 7 \\
 & 5x^3 - 5x^2 \\
 & - \quad + \\
\hline
 & \quad -3x^2 + 5x \\
 & \quad -3x^2 + 3x \\
 & \quad + \quad - \\
\hline
 & \qquad 2x - 7 \\
 & \qquad 2x - 2 \\
 & \qquad - \quad + \\
\hline
 & \qquad\quad -5
\end{array}
$$

(i) $\frac{5x^3}{x} = 5x^2$

(ii) $-\frac{3x^2}{x} = -3x$

(iii) $\frac{2x}{x} = 2$

Quotient $= 5x^2 - 3x + 2$, Remainder $= -5$

2. $\mathbf{(2x^2 - 3x - 14) \div (x + 2)}$

$$
\begin{array}{r|l}
 & 2x - 7 \\
\hline
x + 2 & 2x^2 - 3x - 14 \\
 & 2x^2 + 4x \\
 & - \quad - \\
\hline
 & \quad -7x - 14 \\
 & \quad -7x - 14 \\
 & \quad + \quad + \\
\hline
 & \qquad 0
\end{array}
$$

(i) $\frac{2x^2}{x} = 2x$

(ii) $-\frac{7x}{x} = -7$

Quotient $= 2x - 7$, Remainder $= 0$

3. $\left(\mathbf{9 + 4x + 5x^2 + 3x^3}\right) \div (\mathbf{x + 1})$

$3x^3 + 5x^2 + 4x + 9 \div (x+1)$

$$
\begin{array}{r|l}
 & 3x^2 + 2x + 2 \\ \hline
x+1 & 3x^3 + 5x^2 + 4x + 9 \\
 & 3x^3 + 3x^2 \\
 & - \quad - \\ \hline
 & \quad 2x^2 + 4x \\
 & \quad 2x^2 + 2x \\
 & \quad - \quad - \\ \hline
 & \qquad 2x + 9 \\
 & \qquad 2x + 2 \\
 & \qquad - \quad - \\ \hline
 & \qquad\quad 7
\end{array}
$$

(i) $\frac{3x^3}{x} = 3x^2$

(ii) $\frac{2x^2}{x} = 2x$

(iii) $\frac{2x}{x} = 2$

Quotient $= 3x^2 + 2x + 2$, Remainder $= 7$

4. $\left(\mathbf{4x^3 - 2x^2 + 6x + 7}\right) \div (\mathbf{3 + 2x})$

$$
\begin{array}{r|l}
 & 2x^2 - 4x + 9 \\ \hline
2x+3 & 4x^3 - 2x^2 + 6x + 7 \\
 & 4x^3 + 6x^2 \\
 & - \quad - \\ \hline
 & \quad -8x^2 + 6x \\
 & \quad -8x^2 - 12x \\
 & \quad + \quad + \\ \hline
 & \qquad 18x + 7 \\
 & \qquad 18x + 27 \\
 & \qquad - \quad - \\ \hline
 & \qquad\quad -20
\end{array}
$$

(i) $\frac{4x^3}{2x} = 2x^2$

(ii) $-\frac{8x^2}{2x} = -4x$

(iii) $\frac{18x}{2x} = 9$

Quotient $= 2x^2 - 4x + 9$, Remainder $= -20$

5. $\boldsymbol{(-18-9x+7x^2)\div(x-2)}$

$7x^2-9x-18\div(x-2)$

$$
\begin{array}{r|l}
 & 7x+5 \\
\hline
x-2 & 7x^2-9x-18 \\
 & 7x^2-14x \\
 & - \quad + \\
\hline
 & \quad\quad 5x-18 \\
 & \quad\quad 5x-10 \\
 & \quad\quad - \quad + \\
\hline
 & \quad\quad\quad\quad -8
\end{array}
$$

(i) $\frac{7x^2}{x}=7x$

(ii) $\frac{5x}{x}=5$

Quotient $=7x+5$, Remainder $=-8$

Exercise 3.4

1. Find the remainder using remainder theorem, when

(i) $\boldsymbol{3x^3+4x^2-5x+8}$ **is divided by** $\boldsymbol{x-1}$

Let $p(x)=3x^3+4x^2-5x+8$. The zero $x-1$ is 1

When $p(x)$ is divided by $x-1$, the remainder is $p(1)$

$p(1)=3(1)^3+4(1)^2-5(1)+8$

$=3+4-5+8$

$=10$

$\therefore$ The remainder is 10

(ii) $\boldsymbol{5x^3+2x^2-6x+12}$ **is divided by** $\boldsymbol{x+2}$

Let $p(x)=5x^3+2x^2-6x+12$. The zero $x+2$ is -2

When $p(x)$ is divided by $x+2$, the remainder is $p(-2)$

$p(-2)=5(-2)^3+2(-2)^2-6(-2)+12$

$=5(-8)+2(4)+12$

$=-40+8+12+12$

$=-8$

$\therefore$ The remainder is -8

(iii) $\boldsymbol{2x^3-4x^2+7x+6}$ **is divided by** $\boldsymbol{x-2}$

Let $p(x)=2x^3-4x^2+7x+6$. The zero $x-2$ is 2

When $p(x)$ is divided by $x-2$, the remainder is $p(2)$

$p(2) = 2(2)^3 - 4(2)^2 + 7(2) + 6$

$= 2(8) - 4(4) + 14 + 6$

$= 16 - 16 + 14 + 6$

$= 20$

$\therefore$ The remainder is 20

(iv) $4x^3 - 3x^2 + 2x - 4$ is divided by $x + 3$

Let $p(x) = 4x^3 - 3x^2 + 2x - 4$. The zero $x + 3$ is -3

When $p(x)$ is divided by $x + 3$, the remainder is $p(-3)$

$p(-3) = 4(-3)^3 - 3(-3)^2 + 2(-3) - 4$

$= 4(-27) - 3(9) - 6 - 4$

$= -108 - 27 - 6 - 4$

$= -145$

$\therefore$ The remainder is -145

(v) $4x^3 - 12x^2 + 11x - 5$ is divided by $2x - 1$

Let $p(x) = 4x^3 - 12x^2 + 11x - 5$. The zero $2x - 1$ is $\frac{1}{2}$ $\left(2x - 1 = 0 \Rightarrow x = \frac{1}{2}\right)$

When $p(x)$ is divided by $x + 3$, the remainder is $p(-3)$

$p(-3) = 4(-3)^3 - 3(-3)^2 + 2(-3) - 4$

$= 4(-27) - 3(9) - 6 - 4$

$= -108 - 27 - 6 - 4$

$= -145$

$\therefore$ The remainder is -145

(vi) $8x^4 + 12x^3 - 2x^2 - 18x + 14$ is divided by $x + 1$

Let $p(x) = 8x^4 + 12x^3 - 2x^2 - 18x + 14$. The zero $x + 1$ is -1

When $p(x)$ is divided by $x + 1$, the remainder is $p(-1)$

$p(-1) = 8(-1)^4 + 12(-1)^3 - 2(-1)^2 - 18(-1) + 14$

$= 8 - 12 - 2 + 18 + 14$

$= 40 - 12 - 2$

$= 26$

$\therefore$ The remainder is 26

(vii) $x^3 - ax^2 - 5x + 2a$ is divided by $x - a$

Let $p(x) = x^3 - ax^2 - 5x + 2a$. The zero $x - a$ is a

When $p(x)$ is divided by $x - a$, the remainder is $p(a)$

$p(a) = a^3 - a(a)^2 - 5(a) + 2a$

$= a^3 - a^3 - 5a + 2a$

$= -3a$

$\therefore$ The remainder is $-3a$

2. **When the polynomial $2x^3 - ax^2 + 9x - 8$ is divided by $x - 3$ the remainder is 28. Find the value of a**

When $p(x) = 2x^3 - ax^2 + 9x - 8$ is divided by $(x - 3)$ the remainder is 28

Given that $p(3) = 28$

$2x^3 - ax^2 + 9x - 8 = 28$

$2(3)^3 - a(3)^2 + 9(3) - 8 = 28$

$2(27) - 9a + 27 - 8 = 28$

$54 - 9a + 27 - 8 - 28 = 0$

$45 - 9a = 0$

$-9a = -45$

$a = 5$

3. **Find the value of m if $x^3 - 6x^2 + mx + 60$ leaves the remainder 2 when divided by $x + 2$**

When $p(x) = x^3 - 6x^2 + mx + 60$ is divided by $(x + 2)$ the remainder is 2

Given that $p(-2) = 2$

$x^3 - 6x^2 + mx + 60 = 2$

$(-2)^3 - 6(-2)^2 + m(-2) + 60 = 2$

$-8 - 6(4) - 2m + 60 = 2$

$-8 - 24 - 2m + 60 - 2 = 0$

$-2m + 26 = 0$

$2m = 26$

$m = 13$

4. **If $(x - 1)$ divides $mx^3 - 2x^2 + 25x - 26$ without remainder find the value of m**

When $p(x) = mx^3 - 2x^2 + 25x - 26$ is divided by $(x - 1)$ the remainder is 0.

$p(1) = m(1)^3 - 2(1)^2 + 25(1) - 26$

$0 = m - 2 + 25 - 26$

$m = 3$

5. **If the polynomials $x^3 + 3x^2 - m$ and $2x^3 - mx + 9$ leave the same remainder when they are divided by $(x - 2)$, find the value of m. Also find the remainder.**

$p(x) = x^3 + 3x^2 - m$ and $q(x) = 2x^3 - mx + 9$

When $p(x)$ is divided by $(x - 2)$ the remainder is $p(2)$

$p(2) = (2)^3 + 3(2)^2 - m$

$= 8 + 12 - m$

$= 20 - m$

When $q(x)$ is divided by $(x-2)$ the remainder is $q(2)$

$q(x) = 2x^3 - mx + 9$

$= 2(2)^3 - 2m + 9$

$= 16 + 9 - 2m$

$= 25 - 2m$

Given that $p(2) = q(2)$

$20 - m = 25 - 2m$

$2m - m = 25 - 20$

$m = 5$

Substituting $m = 5$ in $p(2)$, we get

$p(2) = (2)^3 + 3(2)^2 - 5$

$= 8 + 3(4) - 5$

$= 8 + 12 - 5$

$= 20 - 5$

$= 15$

Exercise 3.5

1. Determine whether $(x+1)$ is a factor of the following polynomials

(i) $6x^4 + 7x^3 - 5x - 4$

(ii) $2x^4 + 9x^3 + 2x^2 + 10x + 15$

(iii) $3x^3 + 8x^2 - 6x - 5$

(iv) $x^3 - 14x^2 + 3x + 12$

Answer :

(i) $6x^4 + 7x^3 - 5x - 4$

$p(x) = 6x^4 + 7x^3 - 5x - 4$. By factor theorem, $(x+1)$ is a factor of $p(x)$ if $p(-1) = 0$

$p(-1) = 6(-1)^4 + 7(-1)^3 - 5(-1) - 4 = 6 - 7 + 5 - 4 = 4 - 4 = 0$

$(x+1)$ is a factor of $6x^4 + 7x^3 - 5x - 4$

(ii) $2x^4 + 9x^3 + 2x^2 + 10x + 15$

$p(x) = 2x^4 + 9x^3 + 2x^2 + 10x + 15$.

By factor theorem, $(x+1)$ is a factor of $p(x)$ if $p(-1) = 0$

$p(-1) = 2(-1)^4 + 9(-1)^3 + 2(-1)^2 + 10(-1) + 15 = 2 - 9 + 2 - 10 + 15 = 0$

$(x+1)$ is a factor of $6x^4 + 7x^3 - 5x - 4$

(iii) $3x^3 + 8x^2 - 6x - 5$

$p(x) = 3x^3 + 8x^2 - 6x - 5$.

By factor theorem, $(x+1)$ is a factor of $p(x)$ if $p(-1) = 0$

$p(-1) = 3(-1)^3 + 8(-1)^2 - 6(-1) - 5 = -3 + 8 + 6 - 5 = 6 \neq 0$

$(x+1)$ is not a factor of $3x^3 + 8x^2 - 6x - 5$

(iv) $x^3 - 14x^2 + 3x + 12$

$p(x) = x^3 - 14x^2 + 3x + 12$.

By factor theorem, $(x+1)$ is a factor of $p(x)$ if $p(-1) = 0$

$p(-1) = (-1)^3 - 14(-1)^2 + 3(-1) + 12 = -1 - 14 - 3 + 12 = -6 \neq 0$

$(x+1)$ is not a factor of $x^3 - 14x^2 + 3x + 12$

2. Determine whether $(x+4)$ is a factor of $x^3 + 3x^2 - 5x + 36$

$p(x) = x^3 + 3x^2 - 5x + 36$

By factor theorem, $(x+4)$ is a factor of $p(x)$ if $p(-4) = 0$

$p(x) = x^3 + 3x^2 - 5x + 36$

$= (-4)^3 + 3(-4)^2 - 5(-4) + 36$

$= -64 + 48 + 20 + 36$

$= -64 + 104$

$= 40 \neq 0$

$(x+4)$ is not a factor of $x^3 + 3x^2 - 5x + 36$

3. Using factor theorem show that $(x-1)$ is a factor of $4x^3 - 6x^2 + 9x - 7$

$p(x) = 4x^3 - 6x^2 + 9x - 7$

By factor theorem, $(x-1)$ is a factor of $p(x)$ if $p(1) = 0$

$p(x) = 4(1)^3 - 6(1)^2 + 9(1) - 7$

$= 4 - 6 + 9 - 7$

$= 13 - 13$

$= 0$

4. Determine whether $(2x+1)$ is a factor fo $4x^3 + 4x^2 - x - 1$

$p(x) = 4x^3 + 4x^2 - x - 1$

By factor theorem, $(2x+1)$ is a factor of $p(x)$ is a factor of $p(x)$ if $p\left(-\frac{1}{2}\right) = 0$. Now,

$p\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 + 4\left(-\frac{1}{2}\right)^2 - \frac{1}{2} - 1$

$= -4\left(\frac{1}{8}\right) + 4\left(\frac{1}{4}\right) + \frac{1}{2} - 1$

$= -\frac{1}{2} + 1 + \frac{1}{2} - 1$

$= 0$

$\therefore (2x+1)$ is a factor of $4x^3 + 4x^2 - x - 1$

5. Determine the value of p if $(x+3)$ is a factor of $x^3-3x^2-px+24$

$p(x)=x^3-3x^2-px+24$. Since $(x+3)$ is a factor of $p(x)$, the remainder $p(-3)=0$.

Now $p(-3)=0$

$$(-3)^3-3(-3)^2-p(-3)+24=0$$
$$-27-3(9)+3p+24=0$$
$$-27-27+3p+24=0$$
$$-54+3p+24=0$$
$$3p=30$$
$$p=10$$

4. Geometry

ONE MARK :

Choose the correct answer

1. If an angle is equal to one third of its supplement, its measure is equal to

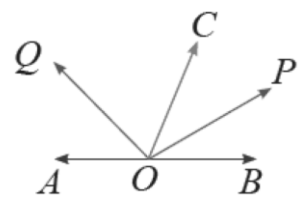
   (a) 40^0 (b) 50^0 **(c) 45^0** (d) 55^0

2. In the given figure, OP bisect, $\angle BOC$ and OQ bisect $\angle AOC$.

   Then $\angle POQ$ is equal to

   (a) 90^0 (b) 120^0

   (c) 60^0 (d) 100^0



3. The complement of an angle exceeds the angle by 60^0.

   Then the angle is equal to

   (a) 25^0 (b) 30^0 **(c) 15^0** (d) 35^0

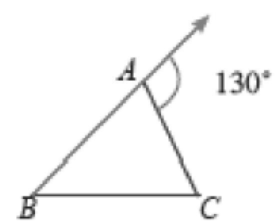
4. Find the measure of an angle, if six times of its complement is 12^0 less than twice of its supplement

   (a) 48^0 (b) 96^0 (c) 24^0 (d) 58^0

5. In the given figure, $\angle B : \angle C = 2:3$, Find $\angle B$

   (a) 120^0 **(b) 52^0**

   (c) 78^0 (d) 130^0



6. $ABCD$ is a parallelogram, E is the midpoint of AB and CE bisects $\angle BCD$.

   Then $\angle DEC$ is

   (a) 60^0 **(b) 90^0** (c) 100^0 (d) 120^0

Exercise 4.1

1. Find the complement of each of the following angles

(i) 63^0 (ii) 24^0 (iii) 48^0 (iv) 35^0 (v) 20^0

Answer :

(i) Complement of $63^0=90^0-63^0=27^0$

(ii) Complement of $24^0 = 90^0 - 24^0 = 66^0$

(iii) Complement of $48^0 = 90^0 - 48^0 = 42^0$

(iv) Complement of $35^0 = 90^0 - 35^0 = 55^0$

(v) Complement of $20^0 = 90^0 - 20^0 = 70^0$

2. Find the supplement of each of the following angles

(i) 58^0 (ii) 148^0 (iii) 120^0 (iv) 40^0 (v) 100^0

Answer :

(i)Supplement of $58^0 = 180^0 - 58^0 = 122^0$

(ii)Supplement of $148^0 = 180^0 - 148^0 = 32^0$

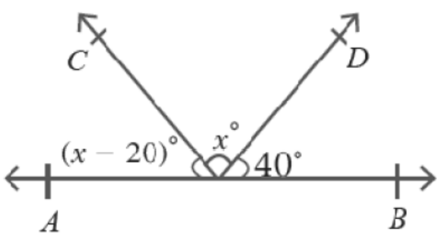
(iii)Supplement of $120^0 = 180^0 - 120^0 = 60^0$

(iv)Supplement of $40^0 = 180^0 - 40^0 = 140^0$

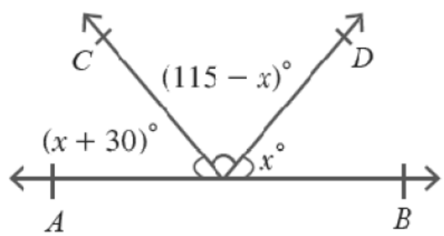
(v)Supplement of $100^0 = 180^0 - 100^0 = 80^0$

3. Find the value of x in the following figures

(i)

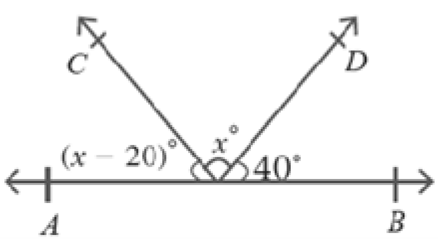


(ii)



Answer :

(i)



$x^0 - 20^0 + x^0 + 40^0 = 180^0$

$2x^0 + 20^0 = 180^0$

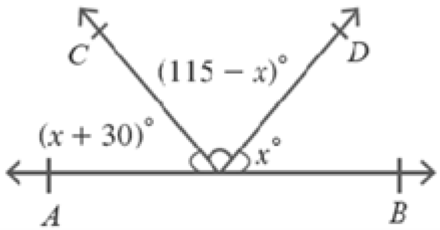
$2x^0 = 180^0 - 20^0 = 160^0$

$x^0 = \frac{160^0}{2} = 80^0$

$x^0 - 20^0 = 80^0 - 20^0 = 60^0$

$x^0 = 80^0$

(ii)



$$x^0 + 30^0 + 115 - x^0 + x^0 = 180^0$$
$$x^0 + 145^0 = 180^0$$
$$x^0 = 180^0 - 145^0 = 35^0$$

4. Find the angles in each of the following

Answer :

(i) The angle which is two times its complement

Angle $= 60^0$, complement angle $= 30^0$, Two times its complement $= 60^0$

(ii) The angle which is four times its supplement

Angle $= 144^0$, Supplement angle $= 36^0$, Four times its supplement $= 144^0$

(iii) The angle whose supplement is four times its complement

Angle $= 60^0$, Supplement angle $= 120^0$, complement $= 30^0$

supplement is four times its complement $(120^0 = 120^0)$

(iv) The angle whose complement is one sixth of its supplement

Angle $= 72^0$, Complement angle $= 18^0$, Supplement angle $= 108^0$

Complement is one sixth of its supplement $(108^0 = 108^0)$

(v) Supplementary angles are in the ratio 4 : 5

Supplementary angles $80^0 + 100^0 = 180^0$

Angles $= 80^0, 100^0$

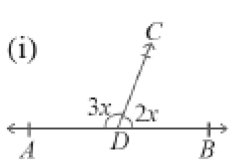
(vi) Two complementary angles are in the ratio 3 : 2

Two complementary angles $54^0, 36^0$

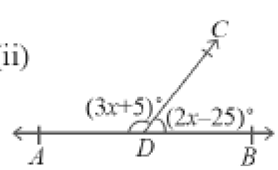
Angles $54^0, 36^0$

5. Find the values of x, y in the following figures

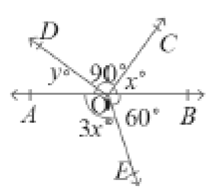
(i)



(ii)

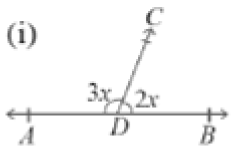


(iii)



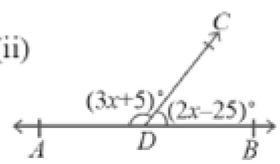
Answer :

(i)



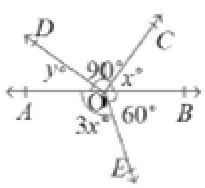
$$3x^0 + 2x^0 = 180^0$$
$$5x^0 = 180^0$$
$$x^0 = 36^0$$

(ii)



$$3x^0 + 5^0 + 2x^0 - 25^0 = 180^0$$
$$5x^0 - 20^0 = 180^0$$
$$5x^0 = 200^0$$
$$x^0 = 40^0$$

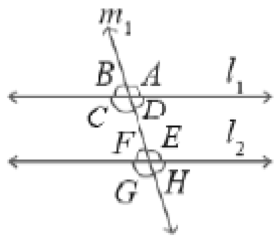
(iii)



$$3x^0 + 60^0 = 180^0$$
$$3x^0 = 120^0$$
$$x^0 = 40^0$$

$$x^0 + y^0 + 90^0 = 180^0$$
$$40^0 + y^0 + 90^0 = 180^0$$
$$y^0 + 130^0 = 180^0$$
$$y^0 = 180^0 - 130^0 = 50^0$$

6. Let $l_1 \parallel l_2$ and m_1 is a transversal. If $\angle F = 65^0$, find the measure of each of the remaining angles



Answer :

Given that $F = 65^0$,

Supplementary angle of F is $E = 180^0 - 65^0 = 115^0$,

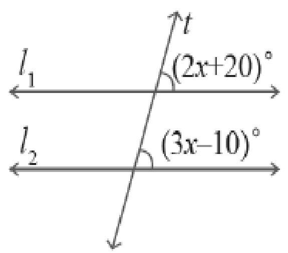
$\therefore$ Vertically opposite angles are equal $\angle A = \angle C = \angle E = \angle G = 115^0$

Supplementary angle of A is $B = 180^0 - 115^0 = 65^0$

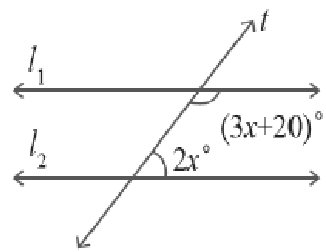
$\therefore$ Vertically opposite angles are equal $\angle B = \angle D = \angle H = 65^0$

7. For what value of x will l_1 and l_2 be parallel lines.

(i)

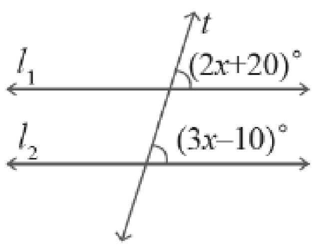


(ii)



Answer :

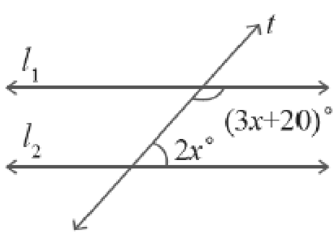
(i)



Corresponding angles are equal $3x^0 - 10^0 = 2x^0 + 20^0$

$$x^0 = 30^0$$

(ii)



Consecutive interior angles are supplementary $3x^0 + 20^0 + 2x^0 = 180^0$

$$5x^0 = 160^0$$

$$3x^0 = 160^0$$

$$x^0 = 32^0$$

8. The angles of a triangle are in the ratio of 1:2:3. Find the mesure if each angle of the triangle.

Let the angle be x^0

Sum of the inner angles of a triangle $= 180^0$

Given that, angles is in the ratio $x^0 : 2x^0 : 3x^0$

$x^0 + 2x^0 + 3x^0 = 180^0$

$6x^0 = 180^0$

$x^0 = 30^0$

$2x^0 = 2(30^0) = 60^0$

$3x^0 = 3(30^0) = 90^0$

9. In $\Delta ABC, \angle A + \angle B = 70^0$ and $\angle B + \angle C = 135^0$. Find the measure of each angle of the triangle.

Given that $\angle A + \angle B = 70^0$, $\angle B + \angle C = 135^0 \Rightarrow \angle C = 135^0 - \angle B$

$\angle A + \angle B + \angle C = 180^0$

$70^0 + 135^0 - \angle B = 180^0$

$205^0 - \angle B = 180^0$

$-\angle B = 180^0 - 205^0$

$-\angle B = -25^0$

$\angle B = 25^0$

$\angle A + \angle B = 70^0$

$\angle A + 25^0 = 70^0$

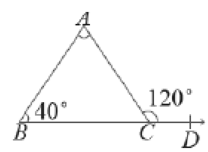
$\angle A = 70^0 - 25^0 = 45^0$

$\angle B + \angle C = 135^0$

$25^0 + \angle C = 135^0$

$\angle C = 135^0 - 25^0 = 110^0$

10. In the given figure at right, side BC of ΔABC is produced to D Find $\angle A$ and $\angle C$.



From the given figure $\angle C + 120^0 = 180^0$

$\angle C = 180^0 - 120^0 = 60^0$

Sum of the inner angles of a triangle $= 180^0$

$\angle A + \angle B + \angle C = 180^0$

$\angle A + 40^0 + 60^0 = 180^0$

$\angle A = 180^0 - 100^0$

$\angle A = 80^0$

Exercise 4.2

1. In a quadrilateral $ABCD$, the angles $\angle A, \angle B, \angle C$ and $\angle D$ are in the ratio 2 : 3: 4: 6. Find the measure of each angle of the quadrilateral.

The sum of the angles of a quadrilateral is 360^0

The angles $\angle A, \angle B, \angle C$ and $\angle D$ are in the ratio 2 : 3: 4: 6

$2x^0 + 3x^0 + 4x^0 + 6x^0 = 360^0$

$15x^0 = 360^0$

$x^0 = 24^0$

$2x^0 = 48^0$

$3x^0 = 72^0$

$4x^0 = 96^0$

$6x^0 =$

2. Suppose $ABCD$ is a parallelogram in which $\angle A = 108^0$. Calculate $\angle B, \angle C$ and $\angle D$

Let $ABCD$ be a parallelogram in which $\angle A = 108^0$

Since $AD \parallel BC$ we can treat AB as a transversal. So,

$\angle A + \angle B = 180^0$

$108^0 + \angle B = 180^0$

$\angle B = 180^0 - 108^0 = 72^0$

Since the opposite angles of a parallelogram are equal, we have,

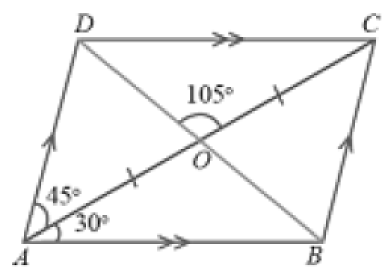
$\angle C = \angle A = 108^0$

$\angle D = \angle B = 72^0$

Hence $\angle B = 72^0, \angle C = 108^0, \angle D = 72^0$

3. In the figure at right, $ABCD$ is a parallelogram $\angle BAO = 30^0, \angle DAO = 45^0$ and $\angle COD = 105^0$

Calculate (i) $\angle ABO$ (ii) $\angle ODC$ (iii) $\angle ACB$ (iv) $\angle CDB$



The diagonals of a parallelogram are equal and bisect each Other.

So, $OA = OB$ and $\angle BAO = \angle OBA = 30^0$

4. Find the measure of each angle of a parallelogram, if larger is 30^0 less than twice the smaller angle

Let the smaller angle be x

Larger angle $= 2x - 30^0$

Sum of the angles $= 180^0$

$\angle A + \angle B = 180^0$

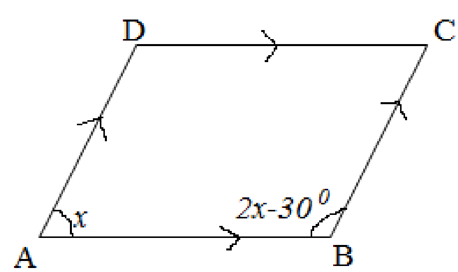
$x + 2x - 30^0 = 180^0$

$3x - 30^0 = 180^0$

$3x = 180^0 + 30^0$

$3x = 210^0$

$x = 70^0$



Smaller angle $= 70^0$

Lager angle$= 2x - 30^0$

$= 2(70^0) - 30^0$

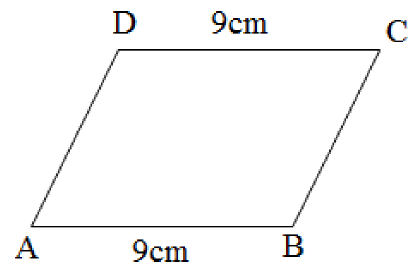
$= 140^0 - 30^0$

$= 110^0$

The measure of each angle of the parallelogram

$$\angle A = 70^0, \angle B = 110^0, \angle C = 70^0, \angle D = 110^0$$

5. Suppose $ABCD$ is a parallelogram in which $AB = 9$ cm and its perimeter is 30 cm. Find the length of each side of the parallelogram.



From the figure, $AB = 9cm$

Perimeter $= 30$ cm

$2(l + b) = 30$

$2(9 + b) = 30$

$9 + b = \frac{30}{2}$

$9 + b = 15$

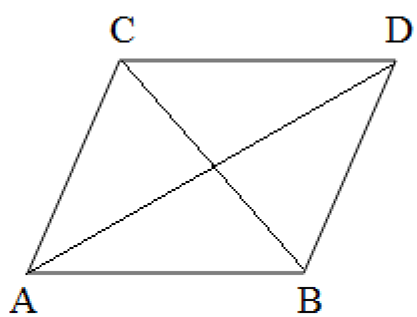
$b = 15 - 9$

$b = 6$ cm

The length of $AB = CD = 9$ cm

Breadth $BC = DA = 6$ cm

6. The length of the diagonals of a rhombus are 24 cm and 18 cm. Find the length of each side of the rhombus.



$d_1 = 24$ cm

$d_2 = 18$ cm

Perimeter of a rhombus $2\sqrt{d_1^2 + d_2^2} = 4a$

$\sqrt{d_1^2 + d_2^2} = \frac{4a}{2}$

$\sqrt{d_1^2 + d_2^2} = 2a$

Squaring on both sides,

$$\left(\sqrt{d_1^2 + d_2^2}\right)^2 = (2a)^2$$

$$d_1^2 + d_2^2 = 4a^2$$

$$(24)^2 + (18)^2 = 4a^2$$

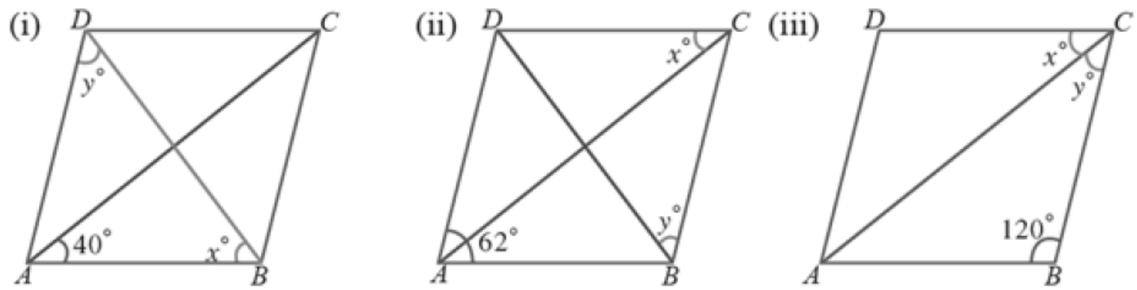
$$576 + 324 = 4a^2$$

$$900 = 4a^2$$

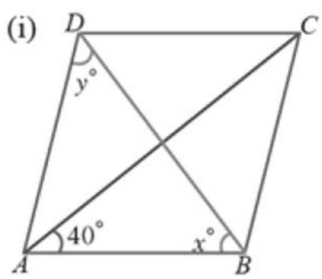
$$a^2 = \frac{900}{4}$$

$$a^2 = 225$$
$$a = \sqrt{225}$$
The length of each side $a = 15$cm

7. In the following figures, $ABCD$ is a rhombus. Find the values of x and y

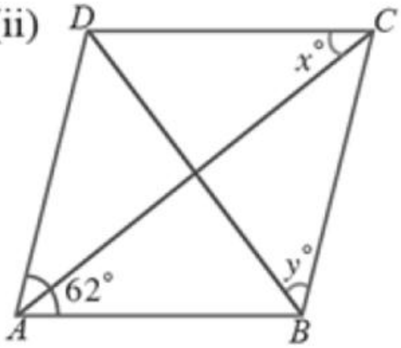


Answer:



In ΔAOB,
$$\angle OAB + \angle ABO + \angle BOA = 180^0$$
$$40^0 + x^0 + 90^0 = 180^0$$
$$130^0 + x = 180^0$$
$$x = 180^0 - 130^0$$
$$x = 50^0$$
In ΔABD,
$$\angle ABD + \angle BDA + \angle DAB = 180^0$$
$$50^0 + y^0 + 40^0 = 180^0$$
$$90 + y^0 = 180^0$$
$$y^0 = 180^0 - 90^0$$
$$y^0 = 90^0$$

(ii)



From the figure $\angle A = \angle C$, $\angle B = \angle D$

$62^0 = \angle C$

Since $CD = CB$

$x^0 = \frac{\angle C}{2} = \frac{62^0}{2} = 31^0$

Sum of the angles $= 360^0$

$\angle A + \angle B + \angle C + \angle D = 360^0$

$62^0 + \angle B + 62^0 + \angle D = 360^0$

$124^0 + \angle B + \angle D = 360^0$

$\angle D + \angle D = 360^0 - 124^0$

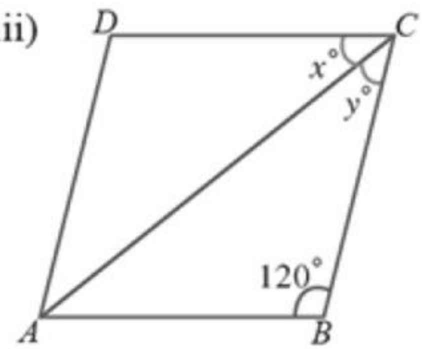
$2\angle D = 236^0$

$\angle D = 118^0$

$\angle B = 118^0$

$\angle y = \frac{\angle B}{2} = \frac{118^0}{2} = 59^0$

(iii)



$\angle A = \angle C$ (Opposite angles)

$\angle B = \angle D$

$\angle B = 120^0$

$\angle D = 120^0$

Sum of the angles $= 360^0$

$\angle A + \angle B + \angle C + \angle D = 360^0$

$\angle A + \angle C + 120^0 + 120^0 = 360^0$

$2\angle A + 240^0 = 360^0$

$2\angle A = 360^0 - 240^0$

$2\angle A = 120^0$

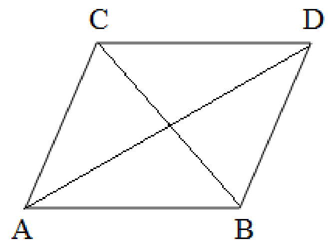
$\angle A = 60^0$

$$\angle C = 60^0$$

$$\angle x + \angle y = 60^0$$

$$\angle x = 30^0, \angle y = 30^0$$

8. The sides of a rhombus is 10 cm and the length of one of the diagonals is 12 cm. Find the length of the other diagonal.



Given, $a = 10$ cm

$d_1 = 12$ cm

$d_2 = ?$

Perimeter of the rhombus $2\sqrt{d_1^2 + d_2^2} = 4a$

$$\sqrt{d_1^2 + d_2^2} = \frac{4a}{2}$$

$$\sqrt{d_1^2 + d_2^2} = 2a$$

Square on both side

$$\left(\sqrt{d_1^2 + d_2^2}\right)^2 = (2a)^2$$

$$d_1^2 + d_2^2 = 4a^2$$

$$144 + d_2^2 = 4(100)$$

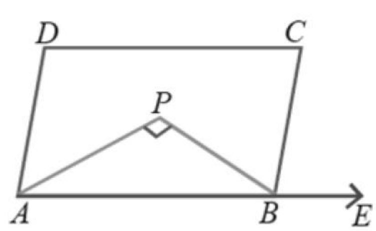
$$d_2^2 = 400 - 144$$

$$d_2^2 = 256$$

$$d_2 = 16$$

The length of other diagonal = 16cm

9. In the figure at the right, $\boldsymbol{ABCD}$ is a parallelogram in which the bisectors of $\angle \boldsymbol{A}$ and $\angle \boldsymbol{B}$ intersect at the point $\boldsymbol{P}$. Prove that $\angle \boldsymbol{APB} = \mathbf{90^0}$



To Prove $\angle APB = 90^0$

$ABCD$ is a parallelogram in which $ABPCD$ and $AD = BC, AB = DC$ (Opposite angles are equal)

Let $\angle CBE = 70^0$

$\angle ABC = 110^0$

In ΔAPB

$\angle APB + \angle PBA + \angle BAP = 180^0$

$\angle APB + 55^0 + 35^0 = 180^0$

$\angle APB + 90^0 = 180^0$

$\angle APB = 180^0 - 90^0$

$\angle APB = 90^0$

5. Coordinate Geometry

1. The point $(-2, 7)$ lies is the quadrant
   (A) I **(B) II** (C) III (D) IV

2. The point $(x, 0)$ where $x < 0$ lies on
   (A) OX (B) OY **(C) $\boldsymbol{OX'}$** (D) OY'

3. For a point $A(a, b)$ lying in quadrant III
   (A) $a > 0, b < 0$ **(B) $\boldsymbol{a} < 0, b < 0$** (C) $a > 0, b > 0$ (D) $a < 0, b > 0$

4. The diagonal of a square formed by the points $(1, 0), (0, 1), (-1, 0)$ and $(0, -1)$ is
   (A) 2 (B) 4 (C)$\sqrt{2}$ (D) 8

5. The triangle obtained by joining the points $A(-5, 0), B(5,0)$ and $C(0, 6)$ is
   (A) an isosceles triangle (B) right triangle
   (C) Scalene triangle (D) an equilateral triangle

6. The distance between the points $(0, 8)$ and $(0, -2)$ is
   (A)6 (B)100 (C)36 **(D)10**

7. $(4, 1), (-2, 1), (7, 1)$ and $(10, 1)$ are points
   (A) on x- axis **(B) on a line parallel to $\boldsymbol{x}$- axis**
   (C) on a line paralled to y- axis (D) on y- axis

8. The distance between the points (a, b) and $(-a, -b)$ is
   (A) $2a$ (B) $2b$ (C) $2a + 2b$ **(D) $\boldsymbol{2\sqrt{a^2 + b^2}}$**

9. The relation between p and q such that the point (p, q) is equidistant from $(-4,0)$ and $(4,0)$
   (A) $\boldsymbol{p = 0}$ (B) $q = 0$ (C) $p + q = 0$ (D) $p + q = 8$

10. The point which is on y- axis with ordinate -5 is
    (A) $\boldsymbol{(0, -5)}$ (B) $(-5, 0)$ (C) $(5, 0)$ (D) $(0, 5)$

Exercise 5.1

1. State whether the following statements are true / false

(i) $(5, 7)$ is a point in the IV quadrant **Answer** : False

(ii) $(-2, -7)$ is a point in the III quadrant **Answer** : True

(iii) $(8,-7)$ lies below the x- axis **Answer** : True

(iv) $(5,2)$ and $(-7,2)$ are points on the line parallel to y axis **Answer** : False

(v) $(-5,2)$ lies to the left of y axis **Answer** : True

(vi) $(0,3)$ is a point on x axis **Answer** : False

(vii) $(-2,3)$ lies in the II quadrant **Answer** : True

(viii) $(-10,0)$ is a point on x- axis **Answer** : True

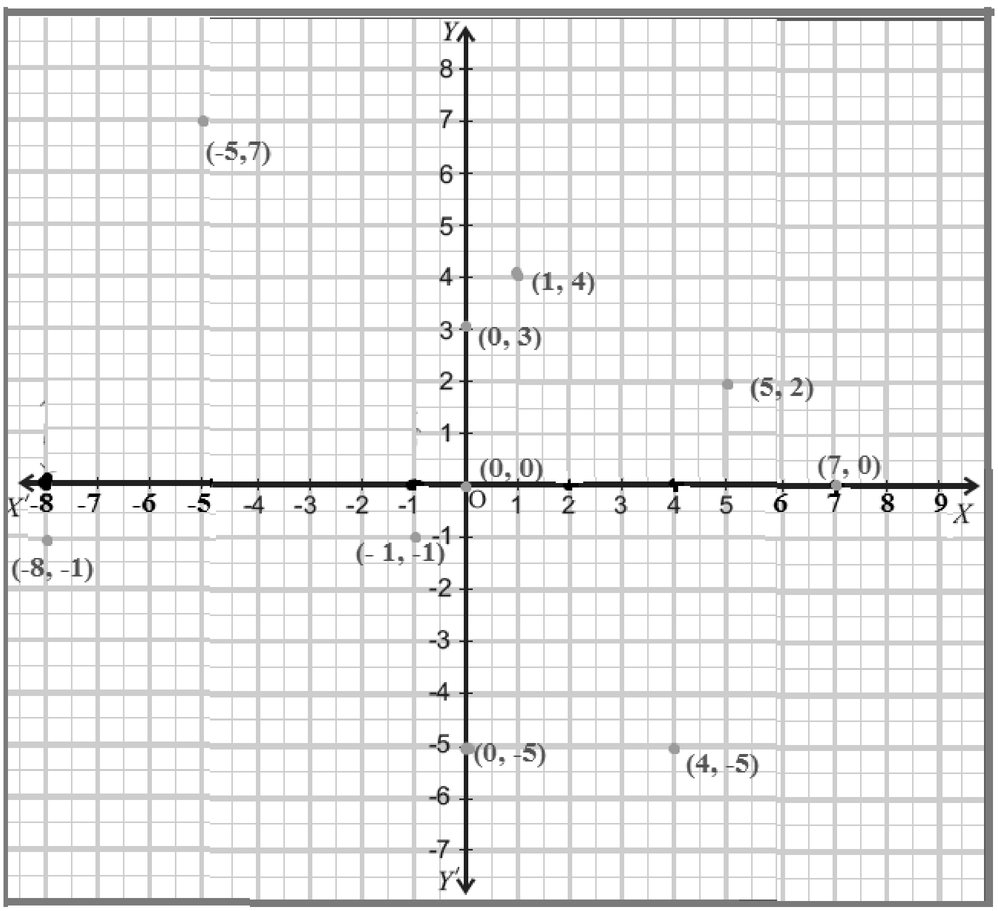
(ix) $(-2\,,-4)$ lies above x- axis **Answer** : False

(x) For any point on the x- axis its y- coordinate is zero **Answer** : True

2. Plot the following points in the coordinate system and specify their quadrant.

(i) $(\mathbf{5},\mathbf{2})$ **(ii)** $(-\mathbf{1},-\mathbf{1})$ **(iii)** $(\mathbf{7},\mathbf{0})$ **(iv)** $(-\mathbf{8},-\mathbf{1})$ **(v)** $(\mathbf{0},-\mathbf{5})$

(vi) $(\mathbf{0},\mathbf{3})$ **(vii)** $(\mathbf{4},-\mathbf{5})$ **(viii)** $(\mathbf{0},\mathbf{0})$ **(ix)** $(\mathbf{1},\mathbf{4})$ **(x)** $(-\mathbf{5},\mathbf{7})$



(i) $(5,2)$ - I quadrant

(ii) $(-1,-1)$ - III quadrant

(iii) $(7,0)$ - on x axis

(iv) $(-8,-1)$ - III quadrant

(v) $(0,-5)$ - on y axis

(vi) $(0,3)$ - on y axis

(vii) $(4,-5)$ - IV quadrant

(viii) $(0,0)$ - Origin

(ix) $(1,4)$ - I quadrant

(x) $(-5,7)$ - II quadrant

3. Write down the abscissa for the following points

(i) $(\mathbf{-7,2})$ **(ii)** $(\mathbf{3,5})$ **(iii)** $(\mathbf{8,-7})$ **(iv)** $(\mathbf{-5,-3})$

Answer :

(i) $(-7,2)$ Abscissa $=-7$

(ii) $(3,5)$ Abscissa$=3$

(iii) $(8,-7)$ Abscissa $=8$

(iv) $(-5,-3)$ Abscissa $=-5$

4. Write down the ordinate of the following points

(i)$(\mathbf{7,5})$ **(ii)**$(\mathbf{2,9})$ **(iii)** $(\mathbf{-5,8})$ **(iv)** $(\mathbf{7,-4})$

Answer :

(i)$(7,5)$ Ordinate $=5$

(ii)$(2,9)$ Ordinate $=9$

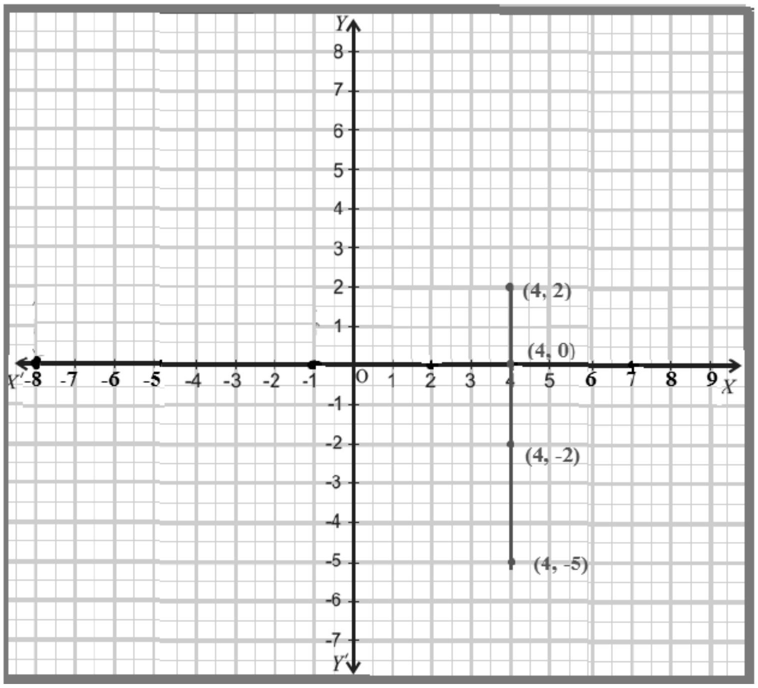
(iii) $(-5,8)$ Ordinate $=8$

(iv) $(7,-4)$ Ordinate $=-4$

5. Plot the following points in the coordinate plane.

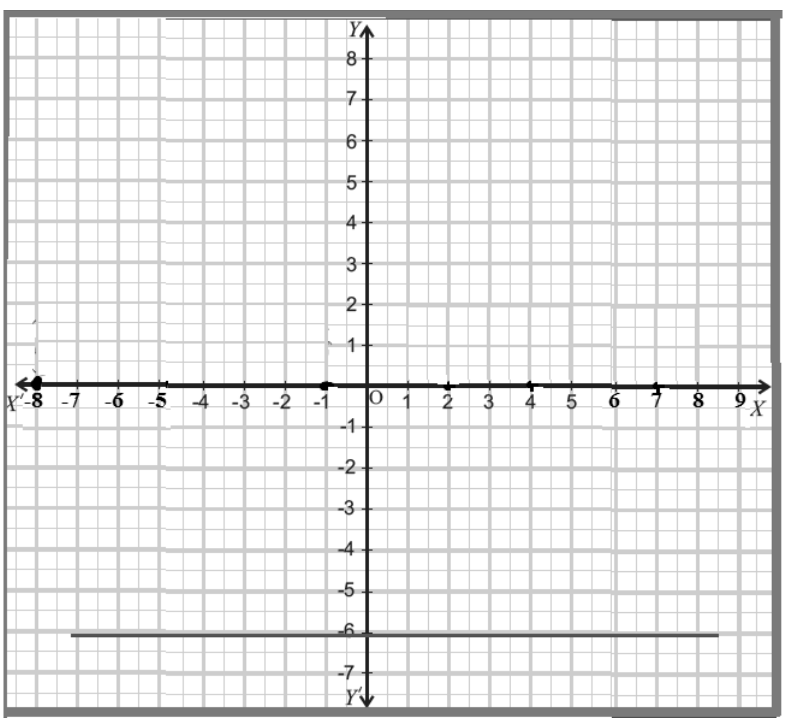
(i) $(\mathbf{4,2})$ **(ii)** $(\mathbf{4,-5})$ **(iii)** $(\mathbf{4,0})$ **(iv)** $(\mathbf{4,-2})$

How is the line joining them situated?



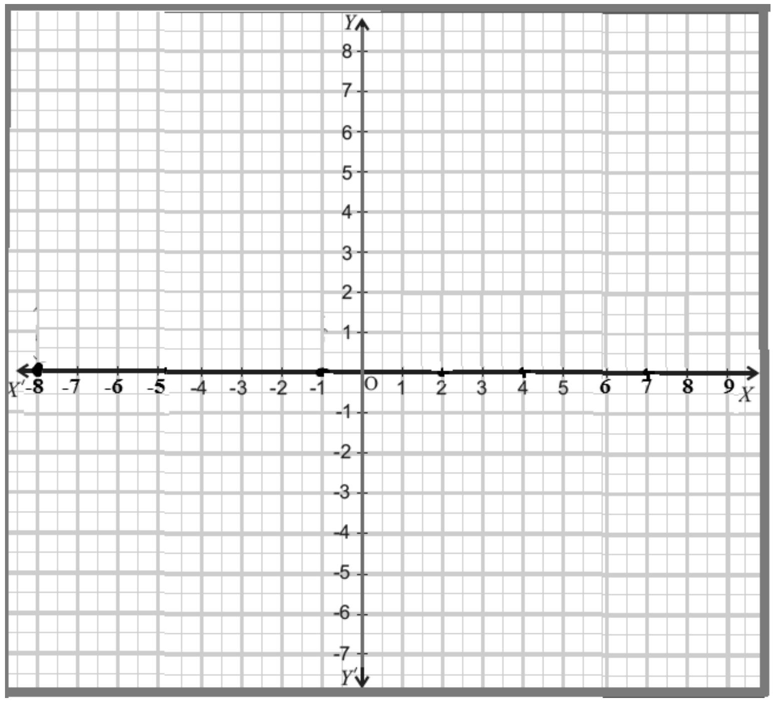
The line joining , situated parallel to y axis

6. The ordinates of two points are each -6. How is the line joining them related with reference to x-axis ?



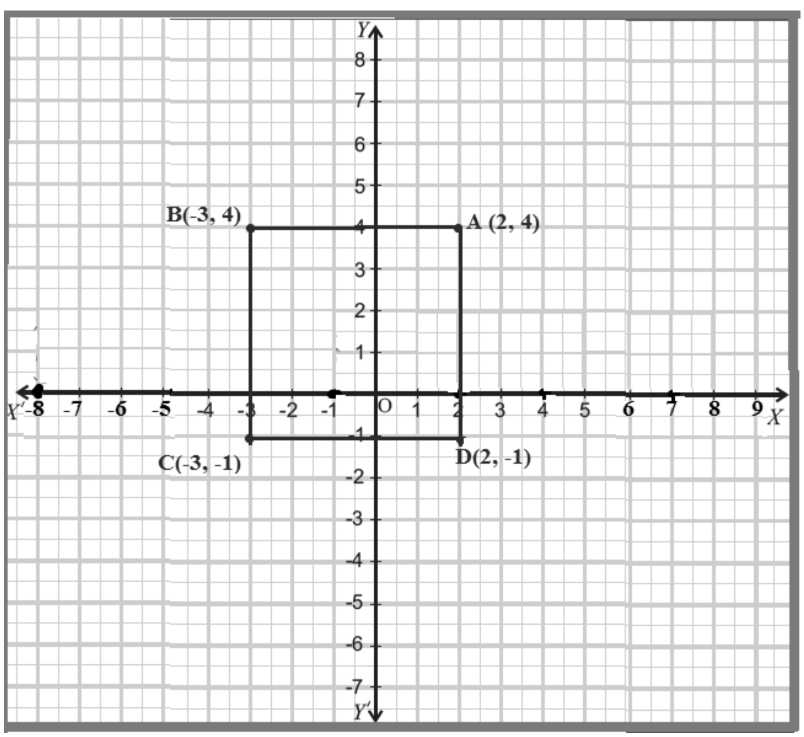
The ordinates of two points are each -6. The line joining them related with reference to parallel to x axis

7. The abscissa of two points is 0. How is the line joining situated?



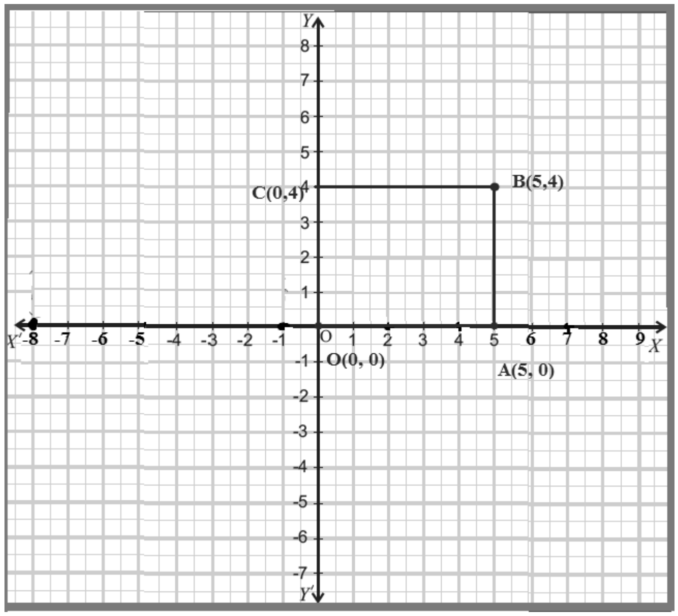
The abscissa of two points is 0. The line joining situated y axis

8. Make the points $A(2,4)$, $B(-3,4)$, $C(-3,-1)$ and $D(2,-1)$ in the cartesian plane. State the figure obtained by joining A and B, B and C, C and D and D and A



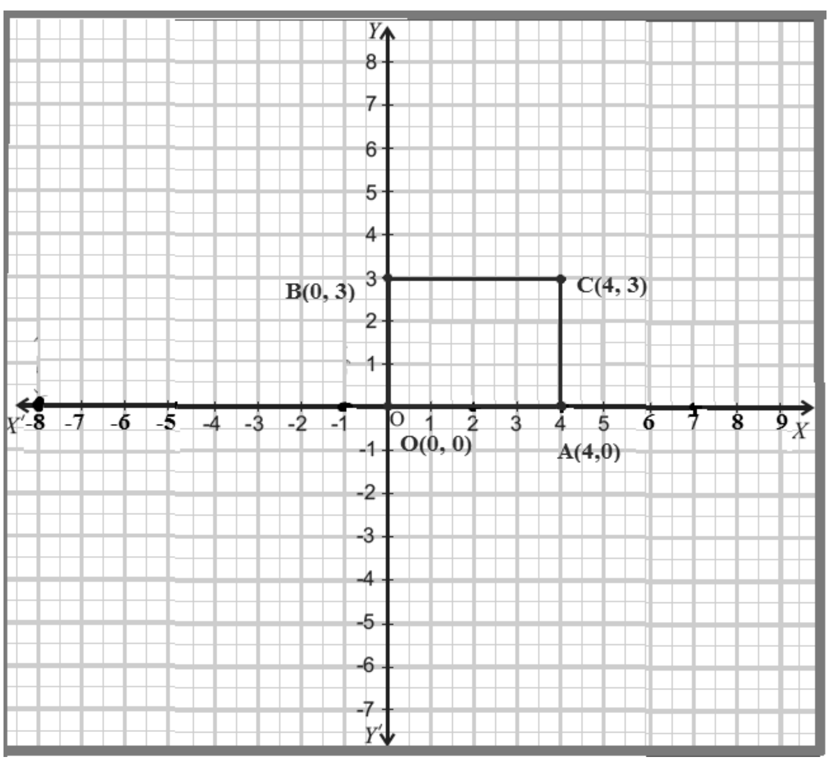
$ABCD$ is a square.

9. **With rectangular axes plot the points $O(0,0)$, $A(5,0)$, $B(5,4)$. Find the coordinate of point C such that $OABC$ forms a rectangle.**



With rectangular axes plot the points $O(0,0)$, $A(5,0)$, $B(5,4)$. The coordinate of point C such that $OABC$ forms a rectangle is $(0, 4)$

10. **In a rectangle $ABCD$, the coordinates of A, B and D are $(0,0)$, $(4,0)$, $(0,3)$. What are the coordinates of C ?**



In a rectangle $ABCD$, the coordinates of A, B and D are $(0,0)$, $(4,0)$, $(0,3)$. The coordinates of $C(4,3)$

Exercise 5.2

1. Find the distance between the following pairs of points

(i) $(\mathbf{7},\mathbf{8})$ **and** $(-\mathbf{2},-\mathbf{3})$ **(ii)** $(\mathbf{6},\mathbf{0})$ **and** $(-\mathbf{2},\mathbf{4})$ **(iii)** $(-\mathbf{3},\mathbf{2})$ **and** $(\mathbf{2},\mathbf{0})$

(iv) $(-\mathbf{2},-\mathbf{8})$ **and** $(-\mathbf{4},-\mathbf{6})$ **(v)** $(-\mathbf{2},-\mathbf{3})$ **and** $(\mathbf{3},\mathbf{2})$ **(v)** $(\mathbf{2},\mathbf{2})$ **and** $(\mathbf{3},\mathbf{2})$

(vii) $(-\mathbf{2},\mathbf{2})$ **and** $(\mathbf{3},\mathbf{2})$ **(viii)** $(\mathbf{7},\mathbf{0})$ **and** $(-\mathbf{8},\mathbf{0})$ **(ix)** $(\mathbf{0},\mathbf{17})$ **and** $(\mathbf{0},-\mathbf{1})$

(x) $(\mathbf{5},\mathbf{7})$ **and the origin**

Answer :

(i) $(\mathbf{7},\mathbf{8})$ **and** $(-\mathbf{2},-\mathbf{3})$

The distance between the points $(7,8)$ and $(-2,-3)$ is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{(-2-7)^2+(-3-8)^2}$$

$$=\sqrt{(-9)^2+(-11)^2}$$

$$=\sqrt{81+121}$$

$$=\sqrt{202}$$

(ii) $(\mathbf{6},\mathbf{0})$ **and** $(-\mathbf{2},\mathbf{4})$

The distance between the points $(6,0)$ and $(-2,4)$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{(-2-6)^2+(4-0)^2}$$

$$=\sqrt{(-8)^2+(4)^2}$$

$$=\sqrt{64+16}$$

$$=\sqrt{80}$$

$$=\sqrt{16\times 5}$$

$$=4\sqrt{5}$$

(iii) $(-\mathbf{3},\mathbf{2})$ **and** $(\mathbf{2},\mathbf{0})$

The distance between the points $(-3,2)$ and $(2,0)$ is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{\left(2-(-3)\right)^2+(0-2)^2}$$

$$=\sqrt{(2+3)^2+(-2)^2}$$

$$=\sqrt{5^2+2^2}$$

$$=\sqrt{25+4}$$

$$=\sqrt{29}$$

(iv) $(\mathbf{-2, -8})$ **and** $(\mathbf{-4, -6})$

The distance between the points $(-2,-8)$ and $(-4,-6)$ is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{\left(-4-(-2)\right)^2+\left(-6-(-8)\right)^2}$$

$$=\sqrt{(-4+2)^2+(-6+8)^2}$$

$$=\sqrt{(-2)^2+(2)^2}$$

$$=\sqrt{4+4}$$

$$=\sqrt{8}$$

$$=2\sqrt{2}$$

(v) $(\mathbf{-2, -3})$ **and** $(\mathbf{3, 2})$

The distance between the points $(-2,-3)$ and $(3,2)$ is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{\left(3-(-2)\right)^2+\left(2-(-3)\right)^2}$$

$$=\sqrt{(5)^2+(5)^2}$$

$$=\sqrt{25+25}$$

$$=\sqrt{2(25)}$$

$$=5\sqrt{2}$$

(vi) $(\mathbf{2, 2})$ **and** $(\mathbf{3, 2})$

The distance between the points $(2,2)$ and $(3,2)$ is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{(3-2)^2+(2-2)^2}$$

$$=\sqrt{1}$$

$$=1$$

(vii) $(\mathbf{-2, 2})$ **and** $(\mathbf{3, 2})$

The distance between the points $(-2,2)$ and $(3,2)$ is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{\left(3-(-2)\right)^2+(2-2)^2}$$

$$=\sqrt{(3+2)^2}$$

$$=\sqrt{5^2}$$

$$=5$$

(viii) $(\mathbf{7},\mathbf{0})$ **and** $(\mathbf{-8},\mathbf{0})$

The distance between the points $(7,0)$ and $(-8,0)$ is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(-8-7)^2+(0-0)^2}$$
$$=\sqrt{(-15)^2}$$
$$=\sqrt{225}$$
$$=15$$

(ix) $(\mathbf{0},\mathbf{17})$ **and** $(\mathbf{0},\mathbf{-1})$

The distance between the points $(0,17)$ and $(0,-1)$ is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(0-0)^2+(-1-17)^2}$$
$$=\sqrt{(-18)^2}$$
$$=18$$

(x) $(\mathbf{5},\mathbf{7})$ **and the origin**

The distance between the points $(5,7)$ and $(0,0)$ is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(0-5)^2+(0-7)^2}$$
$$=\sqrt{(-5)^2+(-7)^2}$$
$$=\sqrt{25+49}$$
$$=\sqrt{74}$$

2. **Show that the following points are collinear**

(i) (3, 7), (6, 5) and $(\mathbf{15},\mathbf{-1})$ **(ii)** $(\mathbf{3},\mathbf{-2})$, $(\mathbf{-2},\mathbf{8})$ **and** $(\mathbf{0},\mathbf{4})$

(iii) $(\mathbf{1},\mathbf{4})$, $(\mathbf{3},\mathbf{-2})$ **and** $(\mathbf{-1},\mathbf{10})$ **(iv)** $(\mathbf{6},\mathbf{2})$, $(\mathbf{2},\mathbf{-3})$ **and** $(\mathbf{-2},\mathbf{-8})$

(v) $(\mathbf{4},\mathbf{1})$, $(\mathbf{5},\mathbf{-2})$ **and** $(\mathbf{6},\mathbf{-5})$

Answer :

(i) (3, 7), (6, 5) and $(\mathbf{15},\mathbf{-1})$

Let the points $A\ (3,7)$, $B(6,5)$ and $C(15,-1)$. By distance formula

$AB^2=(6-3)^2+(5-7)^2=3^2+2^2=9+4=13$

$BC^2=(15-6)^2+(-1-5)^2=9^2+6^2=81+36=117$

$CA^2=(3-15)^2+\left(7-(-1)\right)^2=(-12)^2+(8)^2=144+64=208$

$AB=\sqrt{13}$

$BC = \sqrt{117} = \sqrt{9 \times 13} = 3\sqrt{13}$

$CA = \sqrt{208} = \sqrt{16 \times 13} = 4\sqrt{13}$

This gives $AB + BC = \sqrt{13} + 3\sqrt{13} = 4\sqrt{13} = CA$

Hence A, B and C are collinear.

(ii) $(3, -2)$, $(-2, 8)$ and $(0, 4)$

Let the points $A\ (3, -2\ )$, $B(-2, 8)$ and $C(0, 4)$. By distance formula

$AB^2 = (-2-3)^2 + (8-(-2))^2 = (-5)^2 + 10^2 = 25 + 100 = 125$

$BC^2 = (0-(-2))^2 + (4-8)^2 = 2^2 + (-4)^2 = 4 + 16 = 20$

$CA^2 = (0-3)^2 + \left(4-(-2)\right)^2 = (-3)^2 + (6)^2 = 9 + 36 = 45$

$AB = \sqrt{125} = 5\sqrt{5}$

$BC = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$

$CA = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

This gives $BC + CA = 2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5} = AB$

Hence A, B and C are collinear.

(iii) $(1, 4)$, $(3, -2)$ and $(-1, 10)$

Let the points $A\ (1,4)$, $B(3, -2)$and $C(-1, 10)$. By distance formula

$AB^2 = (3-1)^2 + (-2-4)^2 = 2^2 + 6^2 = 4 + 36 = 40$

$BC^2 = (-1-3)^2 + (10+2)^2 = 4^2 + 12^2 = 16 + 144 = 160$

$CA^2 = (1+1)^2 + (4-10)^2 = (2)^2 + (-6)^2 = 4 + 36 = 40$

$AB = \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$

$BC = \sqrt{160} = \sqrt{16 \times 10} = 4\sqrt{10}$

$CA = \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$

This gives $AB + CA = 2\sqrt{10} + 2\sqrt{10} = 4\sqrt{10} = BC$

Hence A, B and C are collinear.

(iv) $(6, 2)$, $(2, -3)$ and $(-2, -8)$

Let the points $A\ (6, 2)$, $B(2, -3)$ and $C(-2, -8)$. By distance formula

$AB^2 = (2-6)^2 + (-3-2)^2 = (-4)^2 + (-5)^2 = 16 + 25 = 41$

$BC^2 = (-2-2)^2 + (-8+3)^2 = 4^2 + 5^2 = 16 + 25 = 41$

$CA^2 = (6+2)^2 + (2+8)^2 = 8^2 + 10^2 = 64 + 100 = 164$

$AB = \sqrt{41}$

$BC = \sqrt{41}$

$CA = \sqrt{164} = \sqrt{4 \times 41} = 2\sqrt{41}$

This gives $AB + BC = \sqrt{41} + \sqrt{41} = 2\sqrt{13} = CA$

Hence A, B and C are collinear.

(v) $(4, 1), (5, -2)$ and $(6, -5)$

Let the points $A\ (4,1), B(5, -2)$ and $C(6, -5)$. By distance formula

$AB^2 = (5-4)^2 + (-2-1)^2 = 1^2 + 3^2 = 1 + 9 = 10$

$BC^2 = (6-5)^2 + (-5+2)^2 = 1^2 + 3^2 = 1 + 9 = 10$

$CA^2 = (4-6)^2 + (1+5)^2 = (-2)^2 + (6)^2 = 4 + 36 = 40$

$AB = \sqrt{10}$

$BC = \sqrt{10}$

$CA = \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$

This gives $AB + BC = \sqrt{10} + \sqrt{10} = 2\sqrt{10} = CA$

Hence A, B and C are collinear.

3. Show that the following points form an isosceles triangle

(i)$(-2, 0), (4, 0)$ and $(1, 3)$

(ii) $(1, -2), (-5, 1)$ and $(1, 4)$

(iii) $(-1, -3), (2, -1)$ and $(-1, 1)$

(iv) $(1, 3), (-3, -5)$ and $(-3, 0)$

(v) $(2, 3), (5, 7)$ and $(1\ , 4)$

Answer :

(i)$(-2, 0), (4, 0)$ and $(1, 3)$

Let the points $A(-2, 0), B(4, 0)$ and $C(1, 3)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (4+2)^2 + (0-0)^2 = 6^2 = 36$

$BC^2 = (1-4)^2 + (3-0)^2 = 3^2 + 3^2 = 9 + 9 = 18$

$CA^2 = (-2-1)^2 + (3-0)^2 = (-3)^2 + (3)^2 = 9 + 9 = 18$

$AB = \sqrt{36} = 6$

$BC = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$

$CA = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$

$BC = CA = 3\sqrt{2}$

∴ Given points form an isosceles triangle

(ii) $(\mathbf{1}, -\mathbf{2}), (-\mathbf{5}, \mathbf{1})$ **and** $(\mathbf{1}, \mathbf{4})$

Let the points $A(1, -2)$, $B(-5,1)$ and $C(1, 4)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (-5 - 1)^2 + (1 + 2)^2 = 6^2 + 3^2 = 36 + 9 = 45$

$BC^2 = (1 + 5)^2 + (4 - 1)^2 = 6^2 + 3^2 = 36 + 9 = 45$

$CA^2 = (1 - 1)^2 + (4 + 2)^2 = (0)^2 + (6)^2 = 0 + 36 = 36$

$AB = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

$BC = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

$CA = \sqrt{36} = 6$

$AB = BC = 3\sqrt{5}$

∴ Given points form an isosceles triangle

(iii) $(-\mathbf{1}, -\mathbf{3}), (\mathbf{2}, -\mathbf{1})$ **and** $(-\mathbf{1}, \mathbf{1})$

Let the points $A(-1, -3)$, $B(2, -1)$ and $C(-1, 1)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (2 + 1)^2 + (-1 + 3)^2 = 3^2 + 2^2 = 9 + 4 = 13$

$BC^2 = (-1 - 2)^2 + (1 + 1)^2 = (-3)^2 + 2^2 = 9 + 4 = 13$

$CA^2 = (-1 + 1)^2 + (1 + 3)^2 = (0)^2 + (4)^2 = 0 + 16 = 16$

$AB = \sqrt{13}$

$BC = \sqrt{13}$

$CA = \sqrt{16} = 4$

$AB = BC = \sqrt{13}$

∴ Given points form an isosceles triangle

(iv) $(\mathbf{1}, \mathbf{3})$**,** $(-\mathbf{3}, -\mathbf{5})$ **and** $(-\mathbf{3}, \mathbf{0})$

Let the points $A(1, 3)$, $B(-3, -5)$ and $C(-3,0)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (-3 - 1)^2 + (-5 - 3)^2 = 4^2 + 8^2 = 16 + 64 = 80$

$BC^2 = (-3 + 3)^2 + (0 + 5)^2 = (0)^2 + 5^2 = 0 + 25 = 25$

$CA^2 = (1+3)^2 + (3-0)^2 = (4)^2 + (3)^2 = 16 + 9 = 25$

$AB = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

$BC = \sqrt{25} = 5$

$CA = \sqrt{25} = 5$

$BC = CA = 5$

$\therefore$ Given points form an isosceles triangle

(v) $(\mathbf{2}, \mathbf{3}), (\mathbf{5}, \mathbf{7})$ and $(\mathbf{1}, \mathbf{4})$

Let the points $A(2,3)$, $B(5,7)$ and $C(1,4)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (5-2)^2 + (7-3)^2 = 3^2 + 4^2 = 9 + 16 = 25$

$BC^2 = (1-5)^2 + (4-7)^2 = (-4)^2 + (-3)^2 = 16 + 9 = 25$

$CA^2 = (2-1)^2 + (4-3)^2 = (1)^2 + (1)^2 = 1 + 1 = 2$

$AB = \sqrt{25} = 5$

$BC = \sqrt{25} = 5$

$CA = \sqrt{2}$

$AB = BC = 5$

$\therefore$ Given points form an isosceles triangle

4. Show that the following points form a right angled triangle

(i) $(\mathbf{2}, -\mathbf{3}), (-\mathbf{6}, -\mathbf{7})$ and $(-\mathbf{8}, -\mathbf{3})$

(ii) $(-\mathbf{11}, \mathbf{13}), (-\mathbf{3}, -\mathbf{1})$ and $(\mathbf{4}, \mathbf{3})$

(iii) $(\mathbf{0}, \mathbf{0}), (\boldsymbol{a}, \mathbf{0})$ and $(\mathbf{0}, \boldsymbol{b})$

(iv) $(\mathbf{10}, \mathbf{0}), (\mathbf{18}, \mathbf{0})$ and $(\mathbf{10}, \mathbf{15})$

(v) $(\mathbf{5}, \mathbf{9}), (\mathbf{5}, \mathbf{16})$ and $(\mathbf{29}, \mathbf{9})$

Answer :

(i) $(\mathbf{2}, -\mathbf{3}), (-\mathbf{6}, -\mathbf{7})$ and $(-\mathbf{8}, -\mathbf{3})$

Let the points $A(2,-3)$, $B(-6,-7)$ and $C(-8,-3)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (-6-2)^2 + (-7+3)^2 = (-8)^2 + (-4)^2 = 64 + 16 = 80$

$BC^2 = (-8+6)^2 + (-3+7)^2 = (-2)^2 + (4)^2 = 4 + 16 = 20$

$CA^2 = (2+8)^2 + (-3+3)^2 = (10)^2 + (0)^2 = 100$

$AB^2 + BC^2 = 80 + 20 = 100 = CA^2$

Hence ABC is a right angled triangle since the square of one side is equal to sum of the squares of the other two sides

(ii) $(\mathbf{-11, 13}), (\mathbf{-3, -1})$ **and** $(\mathbf{4, 3})$

Let the points $A(-11,13)$, $B(-3,-1)$ and $C(4,3)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (-3+11)^2 + (-1-13)^2 = (8)^2 + (-14)^2 = 64 + 196 = 260$

$BC^2 = (4+3)^2 + (3+1)^2 = (7)^2 + (4)^2 = 49 + 16 = 65$

$CA^2 = (-11-4)^2 + (3-13)^2 = (-15)^2 + (-10)^2 = 225 + 100 = 325$

$AB^2 + BC^2 = 260 + 65 = 325 = CA^2$

Hence ABC is a right angled triangle since the square of one side is equal to sum of the squares of the other two sides

(iii) $(\mathbf{0, 0}), (\boldsymbol{a}, \mathbf{0})$ **and** $(\mathbf{0}, \boldsymbol{b})$

Let the points $A(0,0)$, $B(a,0)$ and $C(0,b)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (a-0)^2 + (0-0)^2 = (a)^2 + (0)^2 = a^2$

$BC^2 = (0-a)^2 + (b-0)^2 = (-a)^2 + (b)^2 = a^2 + b^2$

$CA^2 = (0-0)^2 + (0-b)^2 = (0)^2 + (-b)^2 = 0 + b^2 = b^2$

$AB^2 + CA^2 = a^2 + b^2 = BC^2$

Hence ABC is a right angled triangle since the square of one side is equal to sum of the squares of the other two sides

(iv) $(\mathbf{10, 0}), (\mathbf{18, 0})$ **and** $(\mathbf{10, 15})$

Let the points $A(10,0)$, $B(18,0)$ and $C(10,15)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (18-10)^2 + (0-0)^2 = (8)^2 + (0)^2 = 64$

$BC^2 = (10-18)^2 + (15-0)^2 = (-8)^2 + (15)^2 = 64 + 225 = 289$

$CA^2 = (10-10)^2 + (0-15)^2 = (0)^2 + (-15)^2 = 225$

$AB^2 + CA^2 = 64 + 225 = 289 = BC^2$

Hence ABC is a right angled triangle since the square of one side is equal to sum of the squares of the other two sides

(v) $(5, 9), (5, 16)$ and $(29, 9)$

Let the points $A(5, 9)$, $B(5, 16)$ and $C(29, 9)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (5 - 5)^2 + (9 - 16)^2 = (0)^2 + (7)^2 = 49$

$BC^2 = (29 - 5)^2 + (9 - 16)^2 = (24)^2 + (7)^2 = 576 + 49 = 625$

$CA^2 = (5 - 29)^2 + (9 - 9)^2 = (-24)^2 + (0)^2 = 576$

$AB^2 + CA^2 = 49 + 576 = 625 = BC^2$

Hence ABC is a right angled triangle since the square of one side is equal to sum of the squares of the other two sides

5. Show that the following points form an equilateral triangle.

(i) $(0, 0), (10, 0)$ and $(5, 5\sqrt{3})$

(ii) $(a, 0), (-a, 0)$ and $(0, a\sqrt{3})$

(iii) $(2, 2), (-2, -2)$ and $(-2\sqrt{3}, 2\sqrt{3})$

(iv) $(\sqrt{3}, 2), (0, 1)$ and $(0, 3)$

Answer :

(i) $(0, 0), (10, 0)$ and $(5, 5\sqrt{3})$

Let the points $A(0,0)$, $B(10,0)$ and $C(5, 5\sqrt{3})$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (10 - 0)^2 + (0 - 0)^2 = (10)^2 + (0)^2 = 100$

$BC^2 = (5 - 10)^2 + \left(5\sqrt{3} - 0\right)^2 = (-5)^2 + (5\sqrt{3})^2 = 25 + 25(3) = 25 + 75 = 100$

$CA^2 = (0 - 5)^2 + \left(0 - 5\sqrt{3}\right)^2 = (-5)^2 + +(5\sqrt{3})^2 = 25 + 25(3) = 25 + 75 = 100$

$AB = BC = CA = 10$

Since all the sides are equal the points form an equilateral triangle.

(ii) $(a, 0), (-a, 0)$ and $(0, a\sqrt{3})$

Let the points $A(a, 0)$, $B(-a, 0)$ and $C(0, a\sqrt{3})$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (-a - a)^2 + (0 - 0)^2 = (2a)^2 + (0)^2 = 4a^2$

$BC^2 = (0 + a)^2 + \left(a\sqrt{3} - 0\right)^2 = (a)^2 + (a\sqrt{3})^2 = a^2 + 3a^2 = 4a^2$

$CA^2 = (a - 0)^2 + \left(0 - a\sqrt{3}\right)^2 = (a)^2 + (a\sqrt{3})^2 = a^2 + 3a^2 = 4a^2$

$AB = BC = CA = 2a$

Since all the sides are equal the points form an equilateral triangle.

(iii) $(\mathbf{2},\mathbf{2}),(-\mathbf{2},-\mathbf{2})$ **and** $(-\mathbf{2}\sqrt{\mathbf{3}},\mathbf{2}\sqrt{\mathbf{3}})$

Let the points $A(0,0)$, $B(10,0)$ and $C(5,5\sqrt{3})$

Using the distance formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, we get

$AB^2=(10-0)^2+(0-0)^2=(10)^2+(0)^2=100$

$BC^2=(5-10)^2+\left(5\sqrt{3}-0\right)^2=(-5)^2+(5\sqrt{3})^2=25+25(3)=25+75=100$

$CA^2=(0-5)^2+\left(0-5\sqrt{3}\right)^2=(-5)^2++(5\sqrt{3})^2=25+25(3)=25+75=100$

$AB=BC=CA=10$

Since all the sides are equal the points form an equilateral triangle.

(iv) $(\sqrt{\mathbf{3}},\mathbf{2}),(\mathbf{0},\mathbf{1})$ **and** $(\mathbf{0},\mathbf{3})$

Let the points $A(\sqrt{3},2)$, $B(0,1)$ and $C(0,3)$

Using the distance formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, we get

$AB^2=\left(0-\sqrt{3}\right)^2+(1-2)^2=(\sqrt{3})^2+(-1)^2=3+1=4$

$BC^2=(0-0)^2+(3-1)^2=(0)^2+(2)^2=0+4=4$

$CA^2=\left(\sqrt{3}-0\right)^2+(2-3)^2=\left(\sqrt{3}\right)^2+(-1)^2=3+1=4$

$AB=BC=CA=2$

Since all the sides are equal the points form an equilateral triangle.

6. **Show that the following points taken in order form the vertices of a parallelogram**

(i) $(-\mathbf{7},-\mathbf{5}),(-\mathbf{4},\mathbf{3}),(\mathbf{5},\mathbf{6})$ **and** $(\mathbf{2},-\mathbf{2})$

(ii) $(\mathbf{9},\mathbf{5}),(\mathbf{6},\mathbf{0}),(-\mathbf{2},-\mathbf{3})$ **and** $(\mathbf{1},\mathbf{2})$

(iii) $(\mathbf{0},\mathbf{0}),(\mathbf{7},\mathbf{3}),(\mathbf{10},\mathbf{6})$ **and** $(\mathbf{3},\mathbf{3})$

(iv) $(-\mathbf{2},\mathbf{5}),(\mathbf{7},\mathbf{1}),(-\mathbf{2},-\mathbf{4})$ **and** $(\mathbf{7},\mathbf{0})$

(v) $(\mathbf{3},-\mathbf{5}),(-\mathbf{5},-\mathbf{4}),(\mathbf{7},\mathbf{10})$ **and** $(\mathbf{15},\mathbf{9})$

Answer :

(i) $(-\mathbf{7},-\mathbf{5}),(-\mathbf{4},\mathbf{3}),(\mathbf{5},\mathbf{6})$ **and** $(\mathbf{2},-\mathbf{2})$

Let the points $A(-7,-5)$, $B(-4,3)$, $C(5,6)$and $D(2,-2)$

Using the distance formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, we get

$AB^2=(-4+7)^2+(3+5)^2=(3)^2+(8)^2=9+64=73$

$BC^2=(5+4)^2+(6-3)^2=(9)^2+(3)^2=81+9=90$

$CD^2 = (2-5)^2 + (-2-6)^2 = (-3)^2 + (-8)^2 = 9 + 64 = 73$

$DA^2 = (-7-2)^2 + (-5+2)^2 = (-9)^2 + (-3)^2 = 81 + 9 = 90$

$AB = \sqrt{73}$,

$BC = \sqrt{90}$,

$CD = \sqrt{73}$,

$DA = \sqrt{90}$

$\therefore AB = CD = \sqrt{73},\ BC = DA = \sqrt{90}$

The opposite sides are equal. Hence $ABCD$ is a parallelogram

(ii) $(\mathbf{9},\mathbf{5}), (\mathbf{6},\mathbf{0}), (-\mathbf{2},-\mathbf{3})$ **and** $(\mathbf{1},\mathbf{2})$

Let the points $A(9,5)$, $B(6,0)$, $C(-2,-3)$and $D(1,2)$

Using the distance formula $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$, we get

$AB^2 = (6-9)^2 + (0-5)^2 = (-3)^2 + (-5)^2 = 9 + 25 = 34$

$BC^2 = (-2-6)^2 + (-3-0)^2 = (-8)^2 + (3)^2 = 64 + 9 = 73$

$CD^2 = (1+2)^2 + (2+3)^2 = (3)^2 + (5)^2 = 9 + 25 = 34$

$DA^2 = (9-1)^2 + (5-2)^2 = (8)^2 + (3)^2 = 64 + 9 = 73$

$AB = \sqrt{34}$,

$BC = \sqrt{73}$,

$CD = \sqrt{34}$,

$DA = \sqrt{73}$

$\therefore AB = CD = \sqrt{34},\ BC = DA = \sqrt{73}$

The opposite sides are equal. Hence $ABCD$ is a parallelogram

(iii) $(\mathbf{0},\mathbf{0}), (\mathbf{7},\mathbf{3}), (\mathbf{10},\mathbf{6})$ **and** $(\mathbf{3},\mathbf{3})$

Let the points $A(0,0)$, $B(7,3)$, $C(10,6)$and $D(3,3)$

Using the distance formula $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$, we get

$AB^2 = (7-0)^2 + (3-0)^2 = (7)^2 + (3)^2 = 49 + 9 = 58$

$BC^2 = (10-7)^2 + (6-3)^2 = (3)^2 + (3)^2 = 9 + 9 = 18$

$CD^2 = (3-10)^2 + (3-6)^2 = (-7)^2 + (-3)^2 = 49 + 9 = 58$

$DA^2 = (0-3)^2 + (0-3)^2 = (3)^2 + (3)^2 = 9 + 9 = 18$

$AB = \sqrt{58}$,

$BC = \sqrt{18}$,

$CD = \sqrt{58}$,

$DA = \sqrt{18}$

$\therefore AB = CD = \sqrt{58},\ BC = DA = \sqrt{18}$

The opposite sides are equal. Hence $ABCD$ is a parallelogram

(iv) $(\mathbf{-2, 5}), (\mathbf{7, 1}), (\mathbf{-2, -4})$ **and** $(\mathbf{7, 0})$

Let the points $A(-2,5)$, $B(7,1)$, $C(-2,-4)$and $D(7,0)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (7+2)^2 + (1-5)^2 = (9)^2 + (-4)^2 = 81 + 16 = 97$

$BC^2 = (-2-7)^2 + (-4-1)^2 = (-9)^2 + (-5)^2 = 81 + 25 = 106$

$CD^2 = (7+2)^2 + (0+4)^2 = (9)^2 + (4)^2 = 81 + 16 = 97$

$DA^2 = (-2-7)^2 + (5-0)^2 = (-9)^2 + (5)^2 = 81 + 25 = 106$

$AB = \sqrt{97}$,

$BC = \sqrt{106}$,

$CD = \sqrt{97}$,

$DA = \sqrt{106}$

$\therefore AB = CD = \sqrt{97},\ BC = DA = \sqrt{106}$

The opposite sides are equal. Hence $ABCD$ is a parallelogram

(v) $(\mathbf{3, -5}), (\mathbf{-5, -4}), (\mathbf{7, 10})$ **and** $(\mathbf{15, 9})$

Let the points $A(3,-5)$, $B(-5,-4)$, $C(7,10)$and $D(15,9)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (-5-3)^2 + (-4+5)^2 = (-8)^2 + (1)^2 = 64 + 1 = 65$

$BC^2 = (7+5)^2 + (10+4)^2 = (12)^2 + (14)^2 = 144 + 196 = 340$

$CD^2 = (15-7)^2 + (9-10)^2 = (8)^2 + (-1)^2 = 64 + 1 = 65$

$DA^2 = (15-3)^2 + (9+5)^2 = (12)^2 + (14)^2 = 144 + 196 = 340$

$AB = \sqrt{65}$,

$BC = \sqrt{340}$,

$CD = \sqrt{65}$,

$DA = \sqrt{340}$

$\therefore AB = CD = \sqrt{65},\ BC = DA = \sqrt{340}$

The opposite sides are equal. Hence $ABCD$ is a parallelogram

7. Show that the following points taken in order form the vertices of a rhombus.

(i) $(\mathbf{0}, \mathbf{0}), (\mathbf{3}, \mathbf{4}), (\mathbf{0}, \mathbf{8})$ **and** $(-\mathbf{3}, \mathbf{4})$

Let the points $A(0,0)$, $B(3,4)$, $C(0,8)$and $D(-3,4)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (3-0)^2 + (4-0)^2 = (3)^2 + (4)^2 = 9 + 16 = 25$

$BC^2 = (0-3)^2 + (8-4)^2 = (-3)^2 + (4)^2 = 9 + 16 = 25$

$CD^2 = (-3-0)^2 + (4-8)^2 = (-3)^2 + (-4)^2 = 9 + 16 = 25$

$DA^2 = (0+3)^2 + (0-4)^2 = (3)^2 + (-4)^2 = 9 + 16 = 25$

$AB = \sqrt{25} = 5,$

$BC = \sqrt{25} = 5,$

$CD = \sqrt{25} = 5$,

$DA = \sqrt{25} = 5$

$\therefore AB = BC = CD = DA = \sqrt{25} = 5$

All sides are equal. Hence $ABCD$ is a rhombus

(ii) $(-\mathbf{4}, -\mathbf{7}), (-\mathbf{1}, \mathbf{2}), (\mathbf{8}, \mathbf{5})$ **and** $(\mathbf{5}, -\mathbf{4})$

Let the points $A(-4,-7)$, $B(-1,2)$, $C(8,5)$and $D(5,-4)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (-1+4)^2 + (2+7)^2 = (3)^2 + (9)^2 = 9 + 81 = 90$

$BC^2 = (8+1)^2 + (5-2)^2 = (9)^2 + (3)^2 = 81 + 9 = 90$

$CD^2 = (5-8)^2 + (-4-5)^2 = (-3)^2 + (-9)^2 = 9 + 81 = 90$

$DA^2 = (-4-5)^2 + (-7+4)^2 = (-9)^2 + (-3)^2 = 81 + 9 = 90$

$AB = \sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10},$

$BC = \sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10},$

$CD = \sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10}$,

$DA = \sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10}$

$\therefore AB = BC = CD = DA = 3\sqrt{10}$

All sides are equal. Hence $ABCD$ is a rhombus

(iii) $(\mathbf{1}, \mathbf{0}), (\mathbf{5}, \mathbf{3}), (\mathbf{2}, \mathbf{7})$ **and** $(-\mathbf{2}, \mathbf{4})$

Let the points $A(1,0)$, $B(5,3)$, $C(2,7)$and $D(-2,4)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$AB^2 = (5-1)^2 + (3-0)^2 = (4)^2 + (3)^2 = 16 + 9 = 25$

$BC^2 = (2-5)^2 + (7-3)^2 = (-3)^2 + (4)^2 = 9 + 16 = 25$

$CD^2 = (-2-2)^2 + (4-7)^2 = (-4)^2 + (3)^2 = 16 + 9 = 25$

$DA^2 = (1+2)^2 + (4-0)^2 = (3)^2 + (4)^2 = 9 + 16 = 25$

$AB = \sqrt{25} = 5,$

$BC = \sqrt{25} = 5,$

$CD = \sqrt{25} = 5\,,$

$DA = \sqrt{25} = 5$

$\therefore AB = BC = CD = DA = \sqrt{25} = 5$

All sides are equal. Hence $ABCD$ is a rhombus

(iv) $(\mathbf{2}, -\mathbf{3}), (\mathbf{6}, \mathbf{5}), (-\mathbf{2}, \mathbf{1})$ **and** $(-\mathbf{6}, -\mathbf{7})$

Let the points $A(2,-3)$, $B(6,5)$, $C(-2,1)$and $D(-6,-7)$

Using the distance formula $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$, we get

$AB^2 = (6-2)^2 + (5+3)^2 = (4)^2 + (8)^2 = 16 + 64 = 80$

$BC^2 = (-2-6)^2 + (1-5)^2 = (-8)^2 + (-4)^2 = 64 + 16 = 80$

$CD^2 = (-6+2)^2 + (-7-1)^2 = (-4)^2 + (-8)^2 = 16 + 64 = 80$

$DA^2 = (2+6)^2 + (-3+7)^2 = (8)^2 + (4)^2 = 64 + 16 = 80$

$AB = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5},$

$BC = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5},$

$CD = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}\,,$

$DA = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

$\therefore\ AB = BC = CD = DA = 4\sqrt{5}$

All sides are equal. Hence $ABCD$ is a rhombus

(v) $(\mathbf{15}, \mathbf{20}), (-\mathbf{3}, \mathbf{12}), (-\mathbf{11}, -\mathbf{6})$ **and** $(\mathbf{7}, \mathbf{2})$

Let the points $A(15{,}20)$, $B(-3, 12)$, $C(-11,-6)$and $D(7{,}2)$

Using the distance formula $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$, we get

$AB^2 = (-3-15)^2 + (12-20)^2 = (-18)^2 + (8)^2 = 324 + 64 = 388$

$BC^2 = (-11+3)^2 + (-6-12)^2 = (-8)^2 + (-18)^2 = 64 + 324 = 388$

$CD^2 = (7+11)^2 + (2+6)^2 = (18)^2 + (8)^2 = 324 + 64 = 388$

$DA^2 = (7+11)^2 + (2+6)^2 = (18)^2 + (8)^2 = 324 + 64 = 388$

$AB = \sqrt{388} = \sqrt{4 \times 97} = 2\sqrt{97},$

$BC = \sqrt{388} = \sqrt{4 \times 97} = 2\sqrt{97}$,

$CD = \sqrt{388} = \sqrt{4 \times 97} = 2\sqrt{97}$,

$DA = \sqrt{388} = \sqrt{4 \times 97} = 2\sqrt{97}$

$\therefore AB = BC = CD = DA = 2\sqrt{97}$

All sides are equal. Hence $ABCD$ is a rhombus

8. Examine whether the following points taken in order form a square

(i) $(\mathbf{0}, -\mathbf{1}), (\mathbf{2}, \mathbf{1}), (\mathbf{0}, \mathbf{3})$ and $(-\mathbf{2}, \mathbf{1})$

(ii) $(\mathbf{5}, \mathbf{2}), (\mathbf{1}, \mathbf{5}), (-\mathbf{2}, \mathbf{1})$ and $(\mathbf{2}, -\mathbf{2})$

(iii) $(\mathbf{3}, \mathbf{2}), (\mathbf{0}, \mathbf{5}), (-\mathbf{3}, \mathbf{2})$ and $(\mathbf{0}, -\mathbf{1})$

(iv) $(\mathbf{12}, \mathbf{9}), (\mathbf{20}, -\mathbf{6}), (\mathbf{5}, -\mathbf{14})$ and $(-\mathbf{3}, \mathbf{1})$

(v) $(-\mathbf{1}, \mathbf{2}), (\mathbf{1}, \mathbf{0}), (\mathbf{3}, \mathbf{2})$ and $(\mathbf{1}, \mathbf{4})$

Answer :

(i) $(\mathbf{0}, -\mathbf{1}), (\mathbf{2}, \mathbf{1}), (\mathbf{0}, \mathbf{3})$ and $(-\mathbf{2}, \mathbf{1})$

Let the vertices be taken as $A(3, -2), B(3, 2), C(-1, 2)$ and $D(-1, -2)$

$AB^2 = (3-3)^2 + (2+2)^2 = 0^2 + 4^2 = 16$

$BC^2 = (-1-3)^2 + (2-2)^2 = (-4)^2 + 0 = 16$

$CD^2 = (-1+1)^2 + (-2-2)^2 = 16$

$DA^2 = (-1-3)^2 + (-2+2)^2 = 16$

$AC^2 = (-1-3)^2 + (2+2)^2 = 4^2 + 4^2 = 16 + 16 = 32$

$BD^2 = (-1-3)^2 + (-2-2)^2 = (-4)^2 + (-4)^2 = 16 + 16 = 32$

$AB = BC = CD = DA = \sqrt{16} = 4$ (That is, all the sides are equal)

$AC = BD = \sqrt{32} = 4\sqrt{2}$ (That is, the diagonals are equal)

Hence the points A, B, C and D form a square.

(ii) $(\mathbf{5}, \mathbf{2}), (\mathbf{1}, \mathbf{5}), (-\mathbf{2}, \mathbf{1})$ and $(\mathbf{2}, -\mathbf{2})$

Let the vertices be taken as $A(5, 2), B(1, 5), C(-2, 1)$ and $D(2, -2)$

$AB^2 = (1-5)^2 + (5-2)^2 = (-4)^2 + 3^2 = 16 + 9 = 25$

$BC^2 = (-2-1)^2 + (1-5)^2 = (-3)^2 + (-4)^2 = 9 + 16 = 25$

$CD^2 = (2+2)^2 + (-2-1)^2 = 4^2 + 3^2 = 16 + 9 = 25$

$DA^2 = (2-5)^2 + (-2-2)^2 = (-3)^2 + (-4)^2 = 9 + 16 = 25$

$AC^2 = (-2-5)^2 + (1-2)^2 = (-7)^2 + 1^2 = 49 + 1 = 50 = \sqrt{25 \times 2} = 5\sqrt{2}$

$BD^2 = (2-1)^2 + (-2-5)^2 = (1)^2 + (-7)^2 = 1 + 49 = 50 = \sqrt{25 \times 2} = 5\sqrt{2}$

$AB = BC = CD = DA = \sqrt{25} = 5$ (That is, all the sides are equal)

$AC = BD = \sqrt{50} = 5\sqrt{2}$ (That is, the diagonals are equal)

Hence the points A, B, C and D form a square.

(iii) $(\mathbf{3}, \mathbf{2}), (\mathbf{0}, \mathbf{5}), (-\mathbf{3}, \mathbf{2})$ **and** $(\mathbf{0}, -\mathbf{1})$

Let the vertices be taken as $A(3,2), B(0,5), C(-3,2)$ and $D(0,-1)$

$AB^2 = (0-3)^2 + (5-2)^2 = (-3)^2 + 3^2 = 9 + 9 = 18$

$BC^2 = (-3-0)^2 + (2-5)^2 = (-3)^2 + (-3)^2 = 9 + 9 = 18$

$CD^2 = (0+3)^2 + (-1-2)^2 = 3^2 + (-3)^2 = 9 + 9 = 18$

$DA^2 = (0-3)^2 + (-1-2)^2 = (-3)^2 + (-3)^2 = 9 + 9 = 18$

$AC^2 = (-3-3)^2 + (2-2)^2 = (-6)^2 + 0^2 = 36$

$BD^2 = (0-0)^2 + (-1-5)^2 = (0)^2 + (-6)^2 = 36$

$AB = BC = CD = DA = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$ (That is, all the sides are equal)

$AC = BD = \sqrt{36} = 6$ (That is, the diagonals are equal)

Hence the points A, B, C and D form a square.

(iv) $(\mathbf{12}, \mathbf{9}), (\mathbf{20}, -\mathbf{6}), (\mathbf{5}, -\mathbf{14})$ **and** $(-\mathbf{3}, \mathbf{1})$

Let the vertices be taken as $A(12,9), B(20,-6), C(5,-14)$ and $D(-3,1)$

$AB^2 = (20-12)^2 + (-6-9)^2 = (8)^2 + (-15)^2 = 64 + 225 = 289$

$BC^2 = (5-20)^2 + (-14+6)^2 = (-15)^2 + (-8)^2 = 225 + 64 = 289$

$CD^2 = (-3-5)^2 + (1+14)^2 = (-8)^2 + (15)^2 = 64 + 225 = 289$

$DA^2 = (12+3)^2 + (9-1)^2 = (15)^2 + (8)^2 = 225 + 64 = 289$

$AC^2 = (5-12)^2 + (-14-9)^2 = (-7)^2 + (-23)^2 = 49 + 529 = 578$

$BD^2 = (-3-20)^2 + (1+6)^2 = (-23)^2 + (7)^2 = 529 + 49 = 578$

$AB = BC = CD = DA = \sqrt{289} = 17$ (That is, all the sides are equal)

$AC = BD = \sqrt{578}$ (That is, the diagonals are equal)

Hence the points A, B, C and D form a square.

(v) $(-\mathbf{1},\mathbf{2}),(\mathbf{1},\mathbf{0}),(\mathbf{3},\mathbf{2})$ **and** $(\mathbf{1},\mathbf{4})$

Let the vertices be taken as $A(-1,2), B(1,0), C(3,2)$ and $D(1,4)$

$AB^2 = (1+1)^2 + (0-2)^2 = (2)^2 + (-2)^2 = 4+4 = 8$

$BC^2 = (3-1)^2 + (2-0)^2 = (2)^2 + (2)^2 = 4+4 = 8$

$CD^2 = (1-3)^2 + (4-2)^2 = (-2)^2 + (2)^2 = 4+4 = 8$

$DA^2 = (-1-1)^2 + (2-4)^2 = (-2)^2 + (-2)^2 = 4+4 = 8$

$AC^2 = (-1-3)^2 + (2-2)^2 = (-4)^2 + (0)^2 = 16$

$BD^2 = (1-1)^2 + (4-0)^2 = (0)^2 + (4)^2 = 16$

$AB = BC = CD = DA = \sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$ (That is, all the sides are equal)

$AC = BD = \sqrt{16} = 4$ (That is, the diagonals are equal)

Hence the points A, B, C and D form a square.

9. **Examine whether the following points taken in order form a rectangle**

(i) $(\mathbf{8},\mathbf{3}),(\mathbf{0},-\mathbf{1}),(-\mathbf{2},\mathbf{3})$ **and** $(\mathbf{6},\mathbf{7})$

(ii) $(-\mathbf{1},\mathbf{1}),(\mathbf{0},\mathbf{0}),(\mathbf{3},\mathbf{3})$ **and** $(\mathbf{2},\mathbf{4})$

(iii) $(-\mathbf{3},\mathbf{0}),(\mathbf{1},-\mathbf{2}),(\mathbf{5},\mathbf{6})$ **and** $(\mathbf{1},\mathbf{8})$

(i) $(\mathbf{8},\mathbf{3}),(\mathbf{0},-\mathbf{1}),(-\mathbf{2},\mathbf{3})$ **and** $(\mathbf{6},\mathbf{7})$

Let the vertices be taken as $A(8,3), B(0,-1), C(-2,3)$ and $D(6,7)$

$AB^2 = (0-8)^2 + (-1-3)^2 = (-8)^2 + (-4)^2 = 64+16 = 80$

$BC^2 = (-2-0)^2 + (3+1)^2 = (-2)^2 + (4)^2 = 4+16 = 20$

$CD^2 = (6+2)^2 + (7-3)^2 = (8)^2 + (4)^2 = 64+16 = 80$

$DA^2 = (6-8)^2 + (7-3)^2 = (-2)^2 + (-4)^2 = 4+16 = 20$

$AB = CD = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$ (That is, opposite sides are equal)

$BC = DA = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$ (That is, opposite sides are equal)

Hence the points A, B, C and D form a square.

(ii) $(-\mathbf{1},\mathbf{1}),(\mathbf{0},\mathbf{0}),(\mathbf{3},\mathbf{3})$ **and** $(\mathbf{2},\mathbf{4})$

Let the vertices be taken as $A(8,3), B(0,-1), C(-2,3)$ and $D(6,7)$

$AB^2 = (0-8)^2 + (-1-3)^2 = (-8)^2 + (-4)^2 = 64+16 = 80$

$BC^2 = (-2-0)^2 + (3+1)^2 = (-2)^2 + (4)^2 = 4+16 = 20$

$CD^2 = (6+2)^2 + (7-3)^2 = (8)^2 + (4)^2 = 64 + 16 = 80$

$DA^2 = (6-8)^2 + (7-3)^2 = (-2)^2 + (-4)^2 = 4 + 16 = 20$

$AB = CD = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$ (That is, opposite sides are equal)

$BC = DA = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$ (That is, opposite sides are equal)

Hence the points A, B, C and D form a square.

(iii) $(\mathbf{-3, 0}), (\mathbf{1, -2}), (\mathbf{5, 6})$ **and** $(\mathbf{1, 8})$

Let the vertices be taken as $A(-3,0), B(1,-2), C(5,6)$ and $D(1,8)$

$AB^2 = (1+3)^2 + (-2-0)^2 = (4)^2 + (-2)^2 = 16 + 4 = 20$

$BC^2 = (5-1)^2 + (6+2)^2 = (4)^2 + (8)^2 = 16 + 64 = 80$

$CD^2 = (1-5)^2 + (8-6)^2 = (-4)^2 + (2)^2 = 16 + 4 = 20$

$DA^2 = (-3-1)^2 + (0-8)^2 = (-4)^2 + (-8)^2 = 16 + 64 = 80$

$AB = CD = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$ (That is, opposite sides are equal)

$BC = DA = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$ (That is, opposite sides are equal)

Hence the points A, B, C and D form a square.

10. If the distance between two points $(\boldsymbol{x}, \mathbf{7})$ and $(\mathbf{1}, \mathbf{15})$ is 10, find $\boldsymbol{x}$.

Distance between two points $(1-x)^2 + (15-7)^2 = 100$

$$1 + x^2 - 2x + 8^2 = 100$$

$$x^2 - 2x = 100 - 64 - 1$$

$$x^2 - 2x = 35$$

$$x^2 - 2x - 35 = 0$$

$$(x-7)(x+5) = 0$$

$$x = 7 \text{ or } x = -5$$

11. Show that (4, 1) is equidistant from the points $(\mathbf{-10}, \mathbf{6})$ and $(\mathbf{9}, \mathbf{-13})$

Let P be the point (4, 1). Let A and B represent the points $(-10,6)$ and $(9,-13)$ respectively.

Since P is equidistant from A and B, we have $PA = PB$.

Using distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$(4+10)^2 + (1-6)^2 = (4-9)^2 + (1+13)^2$$

$$14^2 + 5^2 = 5^2 + 14^2$$

$$196 + 25 = 196 + 25$$

$$221 = 221$$

P is equidistant from the points A and B

12. If two points $(2,3)$ and $(-6,-5)$ are equidistant from the point (x,y), show that $x+y+3=0$

Let P be the point (x,y). Let A and B represent the points (2,3) and $(-6,-5)$ respectively.

Since P is equidistant from A and B, we have $PA = PB$.

Using distance formula $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$(x-2)^2+(y-3)^2 = (x+6)^2+(y+5)^2$$

$$x^2+4-4x+y^2+9-6y = x^2+36+12x+y^2+25+10y$$

$$x^2+4-4x+y^2+9-6y-x^2-36-12x-y^2-25-10y = 0$$

$$-16x-16y-48 = 0$$

$$16x+16y+48 = 0$$

$$\div 16 \qquad x+y=3$$

13. If the length of the line segment with end points $(2,-6)$ and $(2,y)$ is 4, find y

Using distance formula $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$4 = \sqrt{(2-2)^2+(y+6)^2}$$

$$4 = \sqrt{(y+6)^2}$$

Squaring on both sides,

$$16 = (y+6)^2$$

$$16 = y^2+36+12y$$

$$y^2+36-16+12y = 0$$

$$y^2+12y+20 = 0$$

$$(y+2)(y+10) = 0$$

$$y=-2, y=-10$$

14. Find the perimeter of the triangle with vertices (i) $(0,8)$, $(6,0)$ and origin

Let A, B, C represents the points (0,8), (6,0), (0,0) respectively

$AB^2 = (6-0)^2+(0-8)^2$

$= 6^2+8^2$

$= 36+64$

$AB^2 = 100$

$AB = 10$

$BC^2 = (0-6)^2+(0-0)^2$

$= 6^2+0$

$= 36$

$BC = 6$

$CA^2 = (0-0)^2+(0-8)^2$

$= 0 + (-8)^2$

$= 64$

$CA = 8$

Perimeter of the triangle $= AB + BC + CA$

$= 10 + 6 + 8$

$= 24$

(ii) $(\mathbf{9}, \mathbf{3}), (\mathbf{1}, -\mathbf{3})$ and origin.

Let A, B, C represents the points $(9,3), (1,-3), (0,0)$ respectively

$AB^2 = (1-9)^2 + (-3-3)^2$

$= 8^2 + 6^2$

$= 64 + 36$

$AB^2 = 100$

$AB = 10$

$BC^2 = (0-1)^2 + (0+3)^2$

$= 1^2 + 3^2 = 1 + 9$

$= 10$

$BC = \sqrt{10}$

$CA^2 = (0-9)^2 + (0-3)^2$

$= 9^2 + 3^2$

$= 81 + 9 = 90$

$CA = \sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10}$

Perimeter of the triangle $= AB + BC + CA$

$= 10 + \sqrt{10} + 3\sqrt{10}$

$= 10 + 4\sqrt{10}$

15. Find the point on the y- axis equidistant from $(-\mathbf{5}, \mathbf{2})$ and $(\mathbf{9}, -\mathbf{2})$ (Hint: A point on the y – axis will have its x-coordinate as zero)

Let the point P be (x, y)

A and B denotes the point $(-5, 2)$ and $(9, -2)$ respectively.

P is equidistant from A and B we have,

$$PA = PB$$

$$PA^2 = PB^2$$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(x+5)^2 + (y-2)^2 = (x-9)^2 + (y+2)^2$

$$x^2 + 25 + 10x + y^2 + 4 - 4y = x^2 + 81 - 18x + y^2 + 4 + 4y$$

$$x^2 + 25 + 10x + y^2 + 4 - 4y - x^2 - 81 + 18x - y^2 - 4 - 4y = 0$$

$$28x - 8y = 56$$

$$\div 4 \quad 7x - 2y = 14$$

A point on the y – axis will have its x-coordinate as zero

Put $x = 0$ $7(0) - 2y = 14$

$$-2y = 14$$

$$y = -7$$

The required point is $P(0, -7)$

16. Find the radius of the circle whose centre is $(3, 2)$ and passes through $(-5, 6)$

Let $C(3,2)$ and $P(-5,6)$

Using distance formula

$$CP^2 = (-5-3)^2 + (6-2)^2$$

$$= (-8)^2 + (4)^2$$

$$= 64 + 16$$

$$= 80$$

$$CP = \sqrt{80}$$

$$= \sqrt{16 \times 5}$$

$$= 4\sqrt{5}$$

$\therefore$ Radius $= 4\sqrt{5}$ units

17. Prove that the points $(0, -5)$, $(4, 3)$ and $(-4, -3)$ lie on the circle centred at the origin with radius 5.

Suppose O represents the points $(0, 0)$

Let the points $(0, -5), (4,3), (-4, -3)$ represent A, B, C respectively.

Using distance formula

$$OA^2 = (0-0)^2 + (-5-0)^2$$

$$= 5^2$$

$$= 25$$

$$OB^2 = (0-4)^2 + (0-3)^2$$

$$= 4^2 + 3^2$$

$$= 16 + 9$$

$$= 25$$

$OC^2 = (0+4)^2 + (0+3)^2$

$= 4^2 + 3^2$

$= 16 + 9$

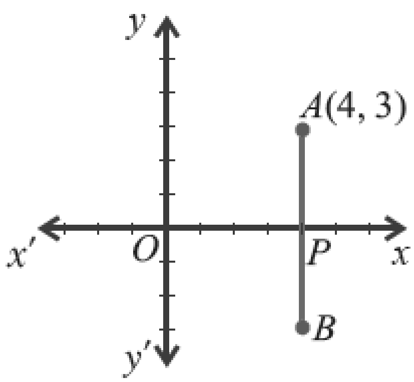
$= 25$

$OA^2 = OB^2 = OC^2 = 25$

$OA = OB = OC = 5$

Hence the points A, B, C are in the circle, with centre $(0,0)$ and its radius is 5 units.

18. **In the figure PB is perpendicular segment from the point $A(4,3)$. If $PA = PB$ then find the coordinates of B.**



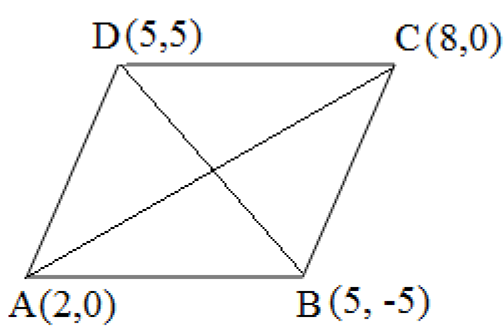
PB is a perpendicular segment from the point $A(4,3)$

$PA = PB$

Then the coordinates of B are $(4, -3)$.

Since it lies in the IV quadrant.

19. **Find the area of the rhombus $ABCD$ with vertices $A(2,0)$, $B(5,-5)$, $C(8,0)$ and $D(5,5)$. [Hint: Area of the rhombus $ABCD = \frac{1}{2}d_1d_2$]**



Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AC^2 = (8-2)^2 + (0-0)^2 + 6^2$

$= 36$

$BD^2 = (5-5)^2 + (-5-5)^2$

$= 0 + 10^2$

$= 100$

$BD = 10$

Area of the Rhombus $ABCD = \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 6 \times 10 = 30$ sq.units

20. Can you draw a triangle with vertices $(\mathbf{1}, \mathbf{5}), (\mathbf{5}, \mathbf{8})$ and $(\mathbf{13}, \mathbf{14})$? Give reason.

$AB = \sqrt{(5-1)^2 + (8-5)^2}$

$= \sqrt{4^2 + 3^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

$BC = \sqrt{(13-5)^2 + (14-8)^2}$

$= \sqrt{8^2 + 6^2}$

$= \sqrt{64+36}$

$= \sqrt{100}$

$= 10$

$CA = \sqrt{(13-1)^2 + (14-5)^2}$

$= \sqrt{12^2 + 9^2}$

$= \sqrt{144+81}$

$= \sqrt{225}$

$= 15$

21. If origin is the centre of a circle with radius 17 units, find the coordinates of any four points on the circle which are not on the axes (Use the Pythagorean triplets)

Suppose O represents the points (0,0). Let A, B, C, D denote the point $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) respectively. The distance of the point (x_1, y_1) from the point (0,0) is

$OA = \sqrt{{x_1}^2 + {y_1}^2}$

Squaring on both sides,

$289 = {x_1}^2 + {y_1}^2$

$64 + 225 = {x_1}^2 + {y_1}^2$

$8^2 + 15^2 = {x_1}^2 + {y_1}^2$

The coordinate of $A(x_1, y_1)$ on the Circle is $(8, -15)$

The coordinate of $B(x_2, y_2)$ on the Circle is $(-8, -15)$

The coordinate of $C(x_3, y_3)$ on the Circle is $(-8, 15)$

The coordinate of $D(x_4, y_4)$ on the Circle is $(8,15)$

22. Show that (2, 1) is the circum-centre of the triangle formed by the vertices $(\mathbf{3}, \mathbf{1})$, (2, 2) and (1, 1)

Circum centre $S(2,1)$

$SA = \sqrt{(3-2)^2 + (1-1)^2}$

$= \sqrt{1^2 + 0}$

$= 1$

$SB = \sqrt{(2-2)^2 + (2-1)^2}$

$= \sqrt{0^2 + 1^2}$

$= 1$

$SC = \sqrt{(1-2)^2 + (1-1)^2}$

$= \sqrt{(-1)^2 + 0^2}$

$= 1$

$SA = SB = SC$

It is known that Circum centre is equidistant from all the vertices of a triangle. Since S is equidistant from the three Vertices, it is the Circum centre of the triangle ABC

23. Show that the origin is the circum-centre of the triangle formed by the vertices (1, 0), $(\mathbf{0}, -\mathbf{1})$ and $\left(-\frac{\mathbf{1}}{\mathbf{2}}, \frac{\sqrt{\mathbf{3}}}{\mathbf{2}}\right)$

Circum centre $S(0,0)$

$SA = \sqrt{(1-0)^2 + (0-0)^2}$

$= \sqrt{1^2 + 0}$

$= 1$

$SB = \sqrt{(0-0)^2 + (-1-0)^2}$

$= \sqrt{0^2 + (-1)^2}$

$= \sqrt{1}$

$= 1$

$SC = \sqrt{\left(-\frac{1}{2}-0\right)^2 + \left(\frac{\sqrt{3}}{2}-0\right)^2}$

$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{1}{4}+\frac{3}{4}}$

$= \sqrt{\frac{4}{4}}$

$= \sqrt{1}$

$= 1$

$SA = SB = SC$

It is known that Circum centre is equidistant from all the vertices of a triangle. Since S is equidistant from the three Vertices, it is the Circum centre of the triangle ABC

24. If the points $A(6,1)$, $B(8,2)$, $C(9,4)$ and $D(p,3)$ taken in order the vertices of a parallelogram, find the value of p using distance formula.

Let $A(6,1)$, $B(8,2)$, $C(9,4)$ and $D(p,3)$.

Using the distance formula

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$AB^2 = (8-6)^2 + (2-1)^2$

$= 2^2 + 1^2 = 4 + 1 = 5$

$BC^2 = (9-8)^2 + (4-2)^2$

$= 1^2 + 2^2 = 1 + 4 = 5$

$CD^2 = (p-9)^2 + (3-4)^2$

$= p^2 + 81 - 18p + (-1)^2$

$= p^2 + 81 - 18p + 1$

$= p^2 + 82 - 18p$

$DA^2 = (p-6)^2 + (3-1)^2$

$= p^2 + 36 - 12p + 2^2$

$= p^2 + 36 - 12p + 4$

$= p^2 + 40 - 12p$

$AB^2 = CD^2$

$5 = p^2 + 82 - 18p$

$p^2 + 77 - 18p = 0$

$p^2 - 18p + 77 = 0$

$(p-7)(p-11)=0$

$p=7,11$

25. **The radius of the circle with centre at the origin is 10 units. Write the coordinates of the point where the circle intersects the axes. Find the distance between any two of such points.**

The Radius of the Circle $= 10$ units

The coordinates of the points where the circle intersect the axes are
$(10,0), (0,10), (-10,0), (0,-10)$

If x_1 and x_2 are the x-Coordinates of two points on the x axis, then the distance between them
$|x_1 - x_2|$

The points on the x axis are $(10,0), (-10,0)$

The distance between the points

$$\begin{aligned} |x_1 - x_2| &= |10-(-10)| \\ &= |10+10| \\ &= |20| \\ &= 20 \end{aligned}$$